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REPORT ON NEW CALCULATIONS OF THE MAGNETIC POTENTIAL OF THE EARTH

Adolf Schmidt

Translation of "Mitteilungen über eine neue Berechnung des Erdmagnetischen Potentials," Abhandlungen der konigl. bayerischen Akademie der Wissenschaften, Munich, Vol. 19, Section 1, 1895, pp 1-66

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16. Abstract

This report concerns an attempt in discovering a procedure to establish better agreement between theoretical and calculated values of the magnetic "potential" of the earth.

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REPORT ON NEW CALCULATIONS OF THE MAGNETIC POTENTIAL OF THE EARTH

Adolf Schmidt
Gotha

I. In the twelfth volume of the publication titled "The Archives of the German Naval Observatory", I provided some mathematical developments on the general theory of the earth's magnetism about 5 years ago to initiate more precise methods applied up to now for studying the magnetic state of the earth and also to supply some preliminary work for the tasks involved in this undertaking. In order to avoid the necessity for research in previous work in order to understand the following report, I will present the contents as briefly as possible. /1*

The attempts previously initiated by the work of Gauss for representing the magnetic force of the earth with an analytical expression have not succeeded in approaching reality, although the observation material has gained consistantly in circumference and reliability. The differences between the observed and calculated values are not essentially smaller than the oldest results attained by Gauss himself, even in the newest calculation of the magnetic potential of the earth, carried out by Neumayer and Petersen. While this gap could be considered a consequence of the lack of an empirical basis in older values, however, it must now be given real significance and the cause should be looked for in insufficient theory, a conclusion first drawn by Mr. Neumayer himself from his studies. An improvement in theory is now possible and necessary in two directions, without taking into consideration the series required for representation. On the one hand, the deviation of the earth from a spherical shape must be taken into consideration; on the other hand, and above all, the two previous assumptions that the magnetic force of the earth exhibits a potential and that the origin of this may be found exclusively in

*Numbers in the margin indicate pagination in the original text.

the earth's core must be rejected.

The first and most important task then resulting is to provide an analytical representation of the distribution of the magnetic force of the earth on the earth's surface, free of that physical hypothesis, a representation which may be calculated with a random degree of accuracy, since it depends only on the reliability of the observed data and on the expansion of the series employed. A sufficient and also most accurate basis is provided with the solution of this purely mathematical task for all further studies with a final goal of providing a physical explanation for the magnetic phenomena of the earth. The question can then be considered on whether the entire force effective at the earth's surface exhibits a potential or to what degree this is not the case, and furthermore, when a potential is found, that portion with an origin outside of the earth can be separated from the portion originating in the earth's core. /4

The flattening of the earth can be taken into consideration with little effort and without including the substantial alterations in calculation compared to those for a sphere. The series developments with a geocentric (not geographical) width or, better still, its complement introduced as an argument,, progress as there according to spherical functions; only certain constant factors are added with the following significance. A first group of these factors depends solely on the flattening of the earth. I have calculated this, employing the Bessel number $1: \alpha = 1:299,1528$ and publish this in the given location (p.12) under the expression p_m^n, π_m^n, q_m^n, x_m^n all the way to the sixth order (i.e. for $0 \leq m$ $n \leq 6$). I compile only the quotients necessary for the present purpose $\pi_m^n : p_m^n$ and $x_m^n : q_m^n$ (Table Ia and Ib). The numbers in these two tables are uncertain by one unit of the sixth decimal position, since the values used in this formation, p_m^n, π_m^n etc. have been rounded off to 6 digits. A second group of factors depends not only on flattening, but also on latitude. These are defined by the equations

$$(1)\quad \alpha = \sqrt{1+\varepsilon^2 \cos v^2} \quad \beta = \sqrt{1+\varepsilon^2} \quad \gamma = \sqrt{1+\varepsilon^2 \cos v^2} : \sqrt{1+\varepsilon^2}$$
$$(\varepsilon^2 = 0{,}00671922)$$

In this, when a designates the radius of the equator and b the polar radius of the earth ellipsoid

$$\varepsilon^2 = (a^2 - b^2) : b^2 = (2\alpha - 1) : (\alpha - 1)^2$$

Furthermore, v is the geocentrical polar distance, connected to the geographical, u, by the known equation /5

$$\operatorname{tg} v = \sqrt{1+\varepsilon^2} \operatorname{tg} u$$

Table II contains, continuing from 5° to 5° toward u, the appropriate values of α, (β), γ, as well as the values of $\alpha \sin v$ and $\beta \sin v$, more important for numerical calculations than α and β themselves. In the calculation of v, a greatly abbreviated approximation equation was employed; therefore, the indicated values deviate from the true values partially by more than 0.5", but always less than 1". After I had employed them once for further calculations, a subsequent alteration, requiring a large amount of correction calculations was not required because of the complete insignificance of those deviations for present purposes.

I have introduced a deviation from the usual procedure in the developments undertaken according to the spherical functions. Following the example given by Gauss, the series were ordinarily developed according to the functions $P^n_m(\cos v) \cos m\lambda$ and $P^n_m(\cos v) \sin m\lambda$ with

$$P_n^m(\cos v) = \sin v^m \left[\cos v^{n-m} - \frac{(n-m)(n-m-1)}{2(2n-1)} \cos v^{n-m-2} + \frac{(n-m)(n-m-1)(n-m-2)(n-m-3)}{2 \cdot 4 \cdot (2n-1)(2n-3)} \cos v^{n-m-4} - \ldots\ldots\right] \tag{2}$$

Meanwhile, the circumstance that these functions are on noticeably different orders of magnitude ($P_1^1, P_1^3, P_1^5, P_1^7$ and therefore also $P^1 \cos\lambda$ etc. achieve, for example, the maximum values 1, 0.275, 0.082, 0.023) leads to significant disturbances in numerical calculations. Above all, the effect of the individual members in the series developed according to spherical functions on the represented value is not proportional to the coefficients, so that it becomes more difficult to rapidly survey the approximate course of function and the significance applied to the individual members. The coefficients multiplied with spherical functions of a higher order, in which m is not completely or almost equal to n, are much too large in comparison to their significance with the others. Therefore, the coefficients must be applied with varying precision since it does not make any sense to employ individual members in an aggregate with substantially more precision than the others. This uncomfortable and rather raw information, /6
since one decimal place already alters the precision of the representation rather considerably, has actually hardly ever been employed. Instead, as far as I know, all coefficients were calculated to the same decimal unit, not justifiable according to the previous argument. Thirdly, it should be remembered that the formation and resolution of equation systems for calculating the series coefficients is facilitated when the factors of this unknown, in this case the function values of P_m^n, differ as little as possible from one another.

The customary development utilized especially in England according to the functions

(3) mit $\mu = \cos v$

$$\text{und } a_0^n = \frac{[1.3.5.\ldots(2n-1)]^2}{n!\, n!}, \quad a_m^n = 2\,\frac{[1.3.5.\ldots(2n-1)]^2}{(n-m)!\,(n+m)!} \text{ für } m > 0$$

$$P_n = \sqrt{a_0^n} \cdot P_0^n, \quad T_n^m = \frac{d^m P_n}{d\mu^m} \sin v^m = \frac{n!}{(n-m)!} \sqrt{a_0^n}\, P_m^n = \sqrt{\frac{(n+m)!}{2(n-m)!}\, a_m^n}\; P_m^n$$

leads to the same distortions, only in reverse and more intensive. The functions P_n all achieve 1 as maximum value, but the T_n^m increase very rapidly with increasing n and m in contrast to the P_m^n. For example

$$T_2^1 = 3\,P_1^2, \quad T_2^2 = 3\,P_2^2, \quad T_6^1 = \frac{693}{8} P_1^6, \qquad T_6^6 = 10395\,P_6^6$$

and the highest values of T_2^1, T_2^2, T_6^1, and T_6^6 are 1.5, 3, 37.1 and 10395. The coefficients of the higher series members are therefore much smaller here than they should be according to their significance.

Because of these considerations, I introduced a certain multiple of P_m^n in place of it in all developments, $R_m^n = r_m^n P_m^n$, chosen in such a manner that the square average value of $R_m^n \cos\ m\lambda$ and $R_m^n \sin\ m\lambda$ achieve the same amount on the entire spherical surface, specifically 1. (It might appear more practical to determine that the average value of R_m should always be equal to 1, where each of the functions $R_m^n \cos\ m\lambda$ and $R_m^n \sin\ m\lambda$ for m greater than 0 would be equal to $\frac{1}{2}\sqrt{2}$ on the average at a power of 2. However, since only practical considerations are to be decisive in the introduction of R_m^n, it appeared better to proceed in the previously mentioned manner, also advantageous for theory, because the members with the factors $R_m^n \cos\ m\lambda$ and $R_m^n \sin\ m\lambda$ always occur independently in the following numerical representations.)

Now, when $d\omega$ designates the surface element of the sphere, /7
in the integration over the entire spherical surface

$$\int (P_m^n(\cos v)\cos m\lambda)^2\, d\omega = \int (P_m^n(\cos v)\sin m\lambda)^2\, d\omega = \frac{4\pi}{(2n+1)a_m^n}$$

This equation applies generally with respect to the first integral but with respect to the second only when m is not 0. The average value of $P_m^n \cos\ m\lambda$ and $P_m^n \sin\ m\lambda$ is therefore $((2n+1)\ a_m)^{-\frac{1}{2}}$, since the spherical surface is 4π. It directly follows that R_m^n attains the desired condition when

$$(4) \qquad R_m^n(\cos v) = r_m^n \cdot P_m^n(\cos v) = \sqrt{(2n+1)\,a_m^n} \cdot P_m^n(\cos v)$$

The values calculated according to this for the factors r_m^n are compiled in Table III, while Table IV contains the values of the functions R_m^n from the first to the seventh order

It should not be overlooked that a complete equality of the functions R_m^n exists only with respect to the average amounts. The maximum values are different; however, the difference is much less than in the case of P_m^n. Under the functions of the first seven orders (with the exception of the constant $R_0^0=1$), R_0^1 and R_1^1 have the smallest maximum in the amount of 1.732, while the largest, i.e. that of R_0^7, amounts to 3.873. The condition that the maximum does not decrease with increasing n in contrast to the functions P_m^n, but that the increase can be termed favorable, because the first, most important members of the development are obtained with somewhat more precision than the remaining values with the same number of decimal places of the coefficients.

It is perhaps not superfluous to demonstrate with an example the alterations established by the introduction of R_m^n in place of P_m^n. For this purpose, I select the development of the magnetic potential of the earth for the time 1885.0 according to the calculations of Neumayer-Petersen, found in the "introductory remarks"

in the fourth section of the Berghaus Physical Atlas (p. 19). According to the remarks on page 5 of my report mentioned at the beginning, I provide all numbers with the opposite sine; moreover, I introduce the quantity $0.1^5 cm^{-\frac{1}{2}} g^{\frac{1}{2}} s^{-1}$. With these alterations, I take the following coefficients from the indicated publication for the series for (V:R) according to the function P_m^n.

/8

n:	1	2	3	4
g_0^n	−31572,0	− 790,6	2436,3	3439,5
g_1^n	− 2481,4	4979,8	−3956,0	3059,7
h_1^n	6025,8	−1299,9	− 738,3	1187,7
g_2^n		566,7	2785,7	1975,4
h_2^n		1260,4	44,3	− 714,7
g_3^n			327,0	− 684,2
h_3^n			549,2	− 512,1
g_4^n				84,9
h_4^n				− 96,8

The coefficients of the development according to the functions R_m^n, on the other hand, are

n:	1	2	3	4
g_0^n	−18228,1	− 235,7	368,8	262,1
g_1^n	− 1432,6	1285,8	− 488,3	184,3
h_1^n	3479,1	− 335,7	− 91,1	71,6
g_2^n		292,6	543,7	168,2
h_2^n		650,9	8,6	− 60,9
g_3^n			156,3	− 109,0
h_3^n			262,6	− 81,6
g_4^n				38,3
h_4^n				− 43,7

After these preliminary explanations provided to avoid a disruptive search for references for the following presentation, I will present the course of calculations after my earlier work in the shortest possible form. In this work, I still assumed the development according to the functions P_m^n; meanwhile, I simply transmit the designations introduced there to the corresponding developments according to the functions R_m^n. The modifications therefore necessary in some equations, limited to the entrance of constant factors dependent on r_m^n, are easily seen; Therefore, I believe that I can introduce them without special emphasis, just as some less important alterations in the designation.

The observed values of the magnetic elements of the earth at the most numerous possible points of the earth's surface form the empirical basis for the entire calculation. First, the components of force X (horizontal to the north), Y (horizontal to the east) and Z (vertically downward) are to be calculated from them, replaced by the quantities αX βY γZ, termed $\overline{X}$, $\overline{Y}$, $\overline{Z}$, deviating only slightly from the first values with respect to the spherical shape of the earth. /9

The first task is now to find an analytical representation for the distribution of the values of $\overline{X}$, $\overline{Y}$, $\overline{Z}$ over the entire surface of the earth. This task can be solved, considered theoretically, with randomly precise approximation of the true condition by presenting the quantities $\overline{X}$ sin v, $\overline{Y}$ sin v and $\overline{Z}$ by rows, progressing according to spherical functions of the arguments v and λ (the geographical length). (In the case of $\overline{X}$ and $\overline{Y}$, this form of representation is not possible, because it becomes variable at the poles. In contrast, however, other expressions, for example, ($\overline{X}$ cosλ +$\overline{Y}$ cos v sinλ) and ($\overline{X}$ sinλ $-$Y cos v cosλ), occur in place of $\overline{X}$ sin v and $\overline{Y}$ sin v.)

From this series found for $\bar{X}$ sin v and $\bar{Y}$ sin v, the functions are now derived

$$U = \int_0^v \bar{X}\, dv, \qquad W = \psi(v) - \int_0^\lambda \bar{Y} \sin v\, d\lambda$$

where ψ (v) designates the portion of U independent of λ. When both are identical, they represent the potential V of magnetism for the earth on the earth's surface with the exception of a constant factor; specifically, V is the common value of bU and bW, where b signifies the polar radius of the earth.

When U and W differ, this difference provides the proof that the magnetic force in the earth's surface does not exhibit any potential, leading to the conclusion of the existence of electrical currents, passing perpendicularly through this surface. The density, i.e., the intensity of these currents in relation to the surface unit, results clearly from the difference (W-U). The manner in which these currents are enclosed inside and outside of the earth's surface, however, remains completely undetermined and therefore the portion involved in the values of U and W, therefore also of $\bar{X}$ and $\bar{Y}$ and those of $\bar{Z}$.

If it is now required that this portion is a minimum, especially that $\bar{Z}$ is never altered, a large portion remains to which a potential is attributed. This is again designated by V. /10

Finally, when the series development of V is combined with that of the vertical components $\bar{Z}$, the potential stemming from the external and internal agents can be separated.

The final result is therefore formed by three functions V_i, V_a and $\bar{i}$ sin v (= $\alpha\beta$ i sin v), precisely expressing the condition of the magnetic field on the earth's surface represented in the series for $\bar{X}$ sin v, $\bar{Y}$ sin v and $\bar{Z}$, therefore also randomly approaching the true values through completion of the observation

data and continuation of the series development. V_i is the potential of magnetic masses or enclosed currents in the earth's core, V_a is that of such agents in the external space, finally i is the intensity of the current passing through the earth's surface perpendicularly, to be comprehended inside and outside as enclosed, so that its effect in this surface itself is a minimum.

The course of the calculations described in the previous section is represented by the following equations.

$$
\begin{aligned}
\bar{X} \sin v &= \Sigma R_m^n (B_m^n \cos m\lambda + C_m^n \sin m\lambda) \\
\bar{Y} \sin v &= \Sigma R_m^n (D_m^n \cos m\lambda + E_m^n \sin m\lambda) \qquad (5) \\
\bar{Z} &= \Sigma R_m^n (j_m^n \cos m\lambda + k_m^n \sin m\lambda)
\end{aligned}
$$

The symbol Σf_m^n stands here and in the following for $\sum_{n=0}^{n=P} \sum_{m=0}^{m=n}$. The development of $\bar{Z}$ is only subject to the condition that j_0^0 must be equal to 0; in contrast, those of $\bar{X}$ sin v and $\bar{Y}$ sin v are connected by a series of conditional equations with the final expression that the horizontal component of the magnetic force of the earth in addition to all the differential quotients at both poles (i.e. for v=0 and v=180°) is finite and has a clearly determined direction, i.e. that the poles are not variable points for them. These conditional equations are, when R_m^n near the northpole (v=0) is equal to $a_m^n \sin v^m$ with the exception of infinitely small quantities of an higher order, therefore near the southpole (v=180°) equal to $(-1)^{n-m} a_m^n \sin v^m$ and when

/11

$$
\begin{aligned}
\Sigma^0 f_m^n &= f_m^m + f_m^{m+2} + \dots \\
\Sigma^1 f_m^{n'} &= f_m^{m+1} + f_m^{m+3} + \dots
\end{aligned}
$$

is introduced:

$$\text{(6)}\qquad \begin{aligned} &\Sigma^0 a^n_m B^n_m - \Sigma^1 a^n_m E^n_m = 0 \qquad && \Sigma^0 a^n_m C^n_m + \Sigma^1 a^n_m D^n_m = 0 \\ &\Sigma^1 a^n_m B^n_m - \Sigma^0 a^n_m E^n_m = 0 \qquad && \Sigma^1 a^n_m C^n_m + \Sigma^0 a^n_m D^n_m = 0 \end{aligned}$$

When m=0, that sum of the two disappears from which the left side of that equation is composed, since $E^n_o = C^n_o = 0$. The function values a^n_m are determined by the equation

$$\text{(7)}\qquad a^n_m = 2^{n-m} \frac{n!\,(n+m)!}{m!\,(2n)!}\, r^n_m$$

Especially

$$a^m_m = r^m_m,\quad a^{m+1}_m = r^{m+1}_m,\quad a^{m+2}_m = \frac{2m+2}{2m+3}\, r^{m+2}_m,\quad a^{m+3}_m = \frac{2m+2}{2m+5}\, r^{m+3}_m$$

(The corresponding data on the values of P^n_m for v=0, mentioned in the above quoted article p. 21, and the equation (14) drawn from these are incorrect with the exception of m=0.)

(The method developed here for the analytical representation of the components of the earth's magnetism can of course be applied to all cases in which a constant function (a vector function) is present, defined clearly everywhere in size and direction, while the usual, known development is employed for a variable scaled quantity. When the direction of the vector falls into the ellipsoid surface everywhere, the developments given in relation to X and Y are sufficient; otherwise, a simple representation of the vertical components according to spherical functions is added as here, generally without the condition $j^0_0=0$.)

The immediate result of the development found for $\bar{Y}$ sin v is

$$W = \psi(v) - (D_0^0 + D_0^1 R_0^1 + D_0^2 R_0^2 + \ldots)\,\lambda + \Sigma R_m^n \left(\frac{1}{m} E_m^n \cos m\lambda - \frac{1}{m} D_m^n \sin m\lambda\right)$$

$$(8) \qquad = \psi(v) - \sin v^2\,\varphi(v)\,\lambda + W_1 \qquad (m > 0)$$

The derivation of U from $\bar{X}$ sin v is not as simple. When

$$\int_0^v \sin v^{m-1}\,dv = \Pi_m$$

is set and a series of new coefficients $\pi_m\,(= \eta_m \cos m\lambda + \zeta_m \sin m\lambda)$, /12
G_m^n, H_m^n is introduced through two equation systems provided in the following, the result is

$$(9) \quad U = (\pi_1 \Pi_1 + \pi_2 \Pi_2 + \ldots) + \Sigma R_m^n (G_m^n \cos m\lambda + H_m^n \sin m\lambda) = f(v, \lambda) + U_0$$

In this case especially

$$\psi(v) = G_0^0 + G_0^1 R_0^1 + G_0^2 R_0^2 + \ldots.$$

In order to obtain the above-mentioned equation systems in a form most suitable for the numerical calculation, I introduced the following designations:

$$(n)_m = \frac{n^2 - m^2}{4n^2 - 1}$$

$$
(10) \qquad \lambda_m^0 = 1, \quad \lambda_m^1 = 1, \quad \lambda_m^p = \lambda_m^{p-2} \frac{(m+p)(m+p-1)_m}{m+p-1}
$$

$$
\lambda_m^p r_m^{m+p} = \mu_m^p, \qquad (m+p-1)\lambda_m^p r_m^{m+p-1} = \nu_m^p
$$

In order to avoid applying the equations twice (for the coefficients of the cosine and for that of the sine of $m\lambda$), I also write

$$
B_m^n \cos m\lambda + C_m^n \sin m\lambda = A_m^n, \qquad G_m^n \cos m\lambda + H_m^n \sin m\lambda = F_m^n
$$

Now the equations are expressed as follows:

$$
(11) \qquad \begin{aligned}
\pi_m &= \mu_m^0 A_m^m + \mu_m^2 A_m^{m+2} + \mu_m^4 A_m^{m+4} + \dots \\
\nu_m^2 F_m^{m+1} &= \mu_m^2 A_m^{m+2} + \mu_m^4 A_m^{m+4} + \dots \\
\nu_m^4 F_m^{m+3} &= \mu_m^4 A_m^{m+4} + \dots \\
& \dots\dots\dots\dots
\end{aligned}
$$

$$
(12) \qquad \begin{aligned}
\nu_m^1 F_m^m &= \mu_m^1 A_m^{m+1} + \mu_m^3 A_m^{m+3} + \mu_m^5 A_m^{m+5} + \dots \\
\nu_m^3 F_m^{m+2} &= \mu_m^3 A_m^{m+3} + \mu_m^5 A_m^{m+5} + \dots \\
\nu_m^5 F_m^{m+4} &= \mu_m^5 A_m^{m+5} + \dots \\
& \dots\dots\dots\dots
\end{aligned}
$$

It can be seen from these equations that $\overline{Y} \sin v$ must be developed just as far, but $\overline{X} \sin v$ must be developed up to the members of the next higher order in order to obtain the development of U and W up to functions of a defined equal order (n).

After U and W have been calculated, on the one hand, the potential results /13

$$V = \frac{b}{2}(U_0 + W_0) = b\Sigma R_m^n (g_m^n \cos m\lambda + h_m^n \sin m\lambda) \tag{13}$$

on the other hand, the current density of the flow passing through the earth's surface vertically

$$i = \frac{1}{4\pi\alpha\beta\, b \sin v} \frac{\partial^2 (W-U)}{\partial v \partial \lambda} \tag{14}$$

The currents directed into the earth are designated positive in this case.

Finally, the coefficients of the potential of inner forces result

$$V_i = b\Sigma R_m^n (c_m^n \cos m\lambda + s_m^n \sin m\lambda) \tag{15}$$

and those originating from external forces

$$V_a = b\Sigma R_m^n (\gamma_m^n \cos m\lambda + \sigma_m^n \sin m\lambda) \tag{16}$$

through the equations

$$(17) \qquad c_m^n = \epsilon_m^n g_m^n - \delta_m^n j_m^n \qquad s_m^n = \epsilon_m^n h_m^n - \delta_m^n k_m^n$$
$$\gamma_m^n = g_m^n - c_m^n \qquad \sigma_m^n = h_m^n - s_m^n$$

in which

$$(18) \qquad \delta_m^n = 1 : \left(n \frac{\pi_m^n}{p_m^n} + (n+1) \frac{\varkappa_m^n}{q_m^n}\right) \text{ und } \epsilon_m^n = n \frac{\pi_m^n}{p_m^n} \delta_m^n$$

are constants dependent on the flattening of the earth. In Tables Ic and Id the values derived from the numbers of Ia and Ib are compiled (the coefficients c,s have been mistakenly exchanged with γ σ on page 23 of the above-mentioned work in the equations presented above).

The final result of the entire calculations are the three series progressing according spherical functions for V_i:b, V_a:b and $\alpha\beta$ bi, with the later remaining behind the two others by one degree in the order of the highest members of the development, as can be easily seen.

From these two series with coefficients completely independent of one another the series for $\overline{X}$ sin v, $\overline{Y}$ sin v and $\overline{Z}$ can be extracted in reverse when considering the regulation established for the derivation of V:b from U and W. This requires no further explanation. To the above compilation of equations, I will add /14
some developments which I noted in the work mentioned above without deriving them, so that I will deal with this briefly here. As is known, it is possible to assume a clearly defined distribution of free magnetism on an internal surface infinitely adjacent to the earth's surface, corresponding within the earth's surface itself and in the entire external space to the same magnetic effects, as are exerted by the actually present agents recognized as cause

of V_i; furthermore, it is possible to replace the external forces with the potential V_a in the earth's surface by defined magnetization of an external surface infinitely close to it, insofar as its effects are noteworthy in the innerspace and at the surface. More important because it is probably of greater significance for the physical explanation of the earth's magnetism is that the same success can be achieved through an arrangement of electrical currents, as is known, filling an infinite adjacent surface parallel to the earth's surface (in reality, instead of the surface a layer of finite thickness must be assumed as in the first case; however, it is easy to see that when the thickness and distance from the earth's surface are only small in comparison to the radius of the earth, the condensation of the entire current caused by simple vertical shifting with suitable reduction in density alters the total effect in an area only by small amounts of higher order.) The distribution of these currents and the magnetic occupation will now be calculated.

The potential of the free magnetism spread in an area (of never infinite large curvature) amounts to the same at every point on both sides of the surface. More precisely, when V_ε designates the potential at a point with a distance from the surface ε calculated positively from a defined direction, $V_{+\varepsilon}$ and $V_{-\varepsilon}$ have the same limit value within one and the same normal with infinitely decreasing ε. This is represented in a frequently applied symbolic manner

$$V_{+0} = V_{-0}$$

The alteration in potential or, practically the same, the component of force perpendicular to the surface, in contrast, is the same in absolute amount on both sides, insofar as it stems from the infinite adjacent portions of the magnetic field, but in opposing directions. The difference of the values on both sides depends on the surface density ρ at the appropriate position. /15

Concerning the effect of the remaining field situated at a finite distance, it supplies the amount of 0 to every difference as constant variable; therefore, this remains unaltered and is dependent upon ρ. Written symbolically this is

$$\left(\frac{\partial V}{\partial n}\right)_{+0} - \left(\frac{\partial V}{\partial n}\right)_{-0} = -4\pi\varrho$$

or in the application to be considered here for the magnetic force in the earth's surface

$$Z_{+0} - Z_{-0} = -4\pi\varrho$$

In this case it applies that the direction of the normal value from the inside to the outside, i.e. from below to above, is positive, the reverse situation from Z.

On the other hand, when the simple, whole surface is subjected to electric current, the magnetic force caused by this is constantly variable when passing through the surface; especially, it is also

$$\left(\frac{\partial V}{\partial n}\right)_{+0} = \left(\frac{\partial V}{\partial n}\right)_{-0} .$$

or

$$Z_{+0} = Z_{-0}$$

On the other hand, the potential at this point is variable. The difference of the values at both sides in infinitely adjacent points of the same normal value is proportional to the total intensity S of that current passing in the surface, including the observed point:

$$V_{+0} - V_{-0} = 4\pi S$$

Those currents passing in a positive direction (counter-clockwise) considered from the side of the positive normal value are set as positive in the calculation of S and the others are set as negative.

The total intensity S is now apparently the same for all points of a current line and it is altered in the transition from such a line to another one by the amount of the current flowing between both. It is therefore

S=constant

the general equation of the current lines, and the total current present between $S=S_1$ and $S=S_2$ has the intensity S_2-S_1. The current density is inversely proportional to the alternating distances of the lines S=constant, and the direction of current is such increasing values of S on the left when viewed from the side of the positive normal values. /16

In the following equations, the developments serving for the definition of the magnetic condition of the earth's surface are introduced, insofar as this exhibits a potential. This is according to equation (15) and (16)

$$V_i = b\,\Sigma R_m^n (c_m^n \cos m\lambda + s_m^n \sin m\lambda) \qquad V_a = b\,\Sigma R_m^n (\gamma_m^n \cos m\lambda + \sigma_m^n \sin m\lambda)$$

or more briefly

$$V_i = b\Sigma J_m^n \qquad V_a = b\Sigma A_m^n$$

V_i can easily be continued in the external space, since it stems from internal forces, V_a into the earth's core for corresponding reasons. However, V_i here and V_a there are first not determined. Accordingly, I set

$$(19) \qquad \begin{aligned} (V_i)_{+\varepsilon} &= b\Sigma J_m^n \left(\frac{r}{b}\right)^{-n-1} & (V_a)_{+\varepsilon} &= b\Sigma B_m^n \left(\frac{r}{b}\right)^{-n-1} \\ (V_i)_{-\varepsilon} &= b\Sigma K_m^n \left(\frac{r}{b}\right)^{n} & (V_a)_{-\varepsilon} &= b\Sigma A_m^n \left(\frac{r}{b}\right)^{n} \end{aligned}$$

In order to simplify the equations, I will drop the continuously repeated indices m and m everywhere in the following (also in the newly introduced quantities M, N, π, p, χ, q, δ, ε), and the result will be no more disturbing than the circumstance that the functions A_m and B_m introduced here temporarily are in agreement in the expression with the coefficients occuring at another point for $\overline{X}$ sin v.

In order to derive Z from V, I must refer to equations (18) and (21) of my earlier work, according to which

$$V = b\Sigma\left(M\left(\frac{r}{b}\right)^{-n-1} + N\left(\frac{r}{b}\right)^{n}\right)$$

the value for Z results

$$Z = \frac{\partial V}{\partial n} = \frac{1}{\gamma} \Sigma \left(-(n+1) \frac{\varkappa}{q} M + n \frac{\pi}{p} N \right)$$

Therefore, in the present case

$$(20) \qquad \begin{aligned} (Z_i)_{+\varepsilon} &= -\frac{1}{\gamma} \Sigma (n+1) \frac{\varkappa}{q} J & \qquad (Z_a)_{+\varepsilon} &= -\frac{1}{\gamma} \Sigma (n+1) \frac{\varkappa}{q} B \\ (Z_i)_{-\varepsilon} &= \frac{1}{\gamma} \Sigma n \frac{\pi}{p} K & \qquad (Z_a)_{-\varepsilon} &= \frac{1}{\gamma} \Sigma n \frac{\pi}{p} A \end{aligned}$$

When $V_{+0}=V_{-0}$, in order to represent the observed forces as the effects of a magnetic surface field on the earth, it follows from (19) (for all indices n, m) /17

$J=K \qquad\qquad A=B$

In retrospect to (18) and (20) at

$$\begin{aligned} -4\pi \varrho_i &= (Z_i)_{+0} - (Z_i)_{-0} & \qquad -4\pi \varrho_a &= (Z_a)_{+0} - (Z_a)_{-0} \\ &= -\frac{1}{\gamma} \Sigma \frac{J}{\delta} & &= -\frac{1}{\gamma} \Sigma \frac{A}{\delta} \end{aligned}$$

On the other hand, when it is necessary to attribute the observed effects to currents identical to $Z_{+0}=Z_{-9}$, the equations are derived from equations (20)

$$-(n+1)\frac{\varkappa}{q} J = n\frac{\pi}{p} K \qquad -(n+1)\frac{\varkappa}{q} B = n\frac{\pi}{p} A$$

and the result from (19) and (18) with a slight conversion:

$$\begin{aligned} 4\pi S_i &= (V_i)_{+0} - (V_i)_{-0} \\ &= b\,\Sigma \frac{J}{\varepsilon} \end{aligned} \qquad \begin{aligned} 4\pi S_a &= (V_a)_{+0} - (V_a)_{-0} \\ &= -b\,\Sigma \frac{A}{1-\varepsilon} \end{aligned}$$

In the equations gained in this manner, expressed as follows in a detailed representation

$$(21)\quad \varrho_i = \frac{1}{4\pi\gamma} \Sigma \frac{1}{\delta_m^n} R_m^n (c_m^n \cos m\lambda + s_m^n \sin m\lambda), \quad \varrho_a = \frac{1}{4\pi\gamma} \Sigma \frac{1}{\delta_m^n} R_m^n (\gamma_m^n \cos m\lambda + \sigma_m^n \sin m\lambda)$$

$$(22)\quad S_i = \frac{b}{4\pi} \Sigma \frac{1}{\varepsilon_m^n} R_m^n (c_m^n \cos m\lambda + s_m^n \sin m\lambda), \quad S_a = -\frac{b}{4\pi} \Sigma \frac{1}{1-\varepsilon_m^n} R_m^n (\gamma_m^n \cos m\lambda + \sigma_m^n \sin m\lambda)$$

the complete solution for the two tasks presented at the beginning of these developments is found.

II. From the beginning of my occupation with the studies /18
presented in the previous section, it has been my understandable wish to apply the theoretical developments to the actual state of the earth's magnetic force. I was strengthened in this desire by the results of an initial experiment undertaken in 1886, because the observations did not appear to agree with the hypothesis of a potential for the only portion of the force under consideration, that is the horizontal portion, but a sudden decision on the chosen basis was impossible because of the relative slight difference in the contradictions. I had based the calculations on the charts of declination and horizontal intensity for 1880.0 published by the German Naval Observatory and already employed previously by Quintus Icilius for a determination of potential, charts with a scale, not permitting an expansion in the removal of the functional values beyond 0.001 $cm^{-\frac{1}{2}}g^{\frac{1}{2}}\ s^{-1}$ at H and 0.1 at δ. When I found time several years later to reassume the task, it had to be my desire to use more precise values for the final calculation. The difficult and time consuming work of obtaining these with sufficient completeness became unnecessary because of a very lucky circumstance, also with respect to the factual basis. Director Neumayer wanted to subject the potential calculations carried out by him together with H. Petersen to a controlled repetition and he requested that I replace his honored associate after the death of Mr. Petersen. I did not delay a moment in accepting this request, providing me with material[1], unique in completeness and critical research -- processed by a researcher in the field for decades in scheduled work with all means of personal influence and collected at a central position such as the German Naval Observatory and prepared with a great deal of experience.

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[1] Precise information is supplied on this by the text "Atlas des Erdmagnetismus" (Department IV of the Physical Atlas by Berghaus). Furthermore, Dr. Neumayer's lecture at the VIII German Conference in Berlin (1889) supplies a good survey, "on the presently available material for research on earth and world magnetism." (Verh d. VIII. d. Geogr.-T.zu Berlin, p. 33ff.)

Mr. Neumayer expressed agreement in my carrying out the new calculations according to the completed theory, providing sufficient indirect check of the earlier calculations as a side result. /19

Mr. Neumayer did not limit himself in supplying me the basis for my study; I also thank him -- and I am glad to be able to present this thanks publically -- in other respects for energetic support for my work. His lively interest in its progress led to an almost uninterrupted scientific correspondence, providing me with good suggestions and the joy of critically founded agreement and encouragement. His repeated support also made it possible for me to have a considerable portion of the numerical calculations carried out by an auxiliary calculator. My father, who had already performed valuable and reliable help on earlier occasions in this manner, assumed large sections of the complicated calculations, probably not mastered by most calculators, and I would also like to thank him at this point, so that I hardly had to carry out half of the calculations in the end. The conclusion of the work would have been even more delayed than was already the case without his assistance.

After I had received the basis for calculation in the summer of 1892, to be reported further (p. 21, 22), I first carried out a preliminary development, limited as were all previously publicized to the members of the four first orders. The results of this work, concluded in the spring of 1893, have already been reported briefly in various publications. At the fall meeting of the "Association of Friends of Astronomy and Cosmic physics" in Muenster (1893), Mr. Neumayer provided some information on this, after I had briefly done this for the physical department of the natural science meeting in Nurnberg at their wish. He had undertaken a more detailed report earlier in an article sent by him to the meteorological congress in Chicago. This article has been lost in the meantime in an unexplainable manner.

The following pages contain the report on the expanded representation proceeding to the members of the sixth order, carried out since then. Because of the condition that this was an initial attempt to find a new way, the main emphasis in the discussion of results has been placed on the question of the tasks resulting for further research and the conditions to be fulfilled in order to promote this aim. When certain practical demands are ascertained as unavoidable in this case, already emphasized for other reasons, it may be permissible to express the hope here that this result will contribute to the acceleration of fulfilling these demands. /20

In the numerical calculations, which I will now discuss, it was above all the aspect of facilitating further repetitions with altered empirical basis the determining factor. For this purpose, all operations were carried out generally, insofar as this was possible, and the observation data employed as late as possible, (this produces the further advantage that the general solutions derived in this manner can also be applied for treating other geophysical problems). Furthermore, for the same purpose the calculation of the values of the components of force following from the analytical derivations were examined for a large number of points, work which only has been completed to a small portion. After this is concluded, it will be sufficient on the basis of new material in repetitions and improvements in the work to introduce the differences of the observed values in comparison to those already described and calculated analytically into the calculations. E. Schering has already described in detail the best method for this purpose. [1]

My initial intention of publishing all results and the complete course of calculation in order to facilitate a detailed check or possible alterations had to be given up for the time being.

[1] Compare K. Schering, "The progress in our knowledge of magnetism in the earth," Geogr. Jahrbuch XV, 1891, p. 143 ff.

I had to limit myself to including first the basis for calculation and the most important results; meanwhile, I hope to find the possibility for a more detailed publication at a later time.

The empirical basis for the derivations in the following is the same, as was already noted, as that for the calculation of potential by Neumayer and Petersen. This consists of the trigonometrical series representations of the components of force X, Y, Z for the 25 equidistant parallel circles of 0°, 5°, 10° ... 60° of the northern and southern latitude, continued up to the members of the fourth order. The coefficients of each of these 75 series are based on the 72 functional values, occuring according to the original charts of Dr. Neumayer in the points of intersection of the corresponding parallel with the merideans of 0°, 5°, 10°.....355° eastern longitude of Greenwich. Therefore, the analytical representation of each component is based only on the values in 1800 points, considered as observed, distributed rather evenly over the earth's surface, although excluding the polar areas. In order to provide a view of these values, I compiled some of them in table V. /21

The series are applied according to the designations of Gauss in the form

$$X = k_0 + k_1 \cos\lambda + K_1 \sin\lambda + \ldots\ldots + K_4 \sin 4\lambda$$
$$Y = l_0 + l_1 \cos\lambda + L_1 \sin\lambda + \ldots\ldots + L_4 \sin 4\lambda$$
$$Z = m_0 + m_1 \cos\lambda + M_1 \sin\lambda + \ldots\ldots + M_4 \sin 4\lambda$$

Their coefficients are compiled in the tables VI a, b, c. The unit employed here, as in all further data is

$$0{,}1^5\ cm^{-\frac{1}{2}}\ g^{\frac{1}{2}}\ s^{-1}$$

i.e. the 10000th portion of the Gaussian unit.

It is now necessary to develop these coefficients or rather (p. 9) the products

$$\alpha k_m^n \sin v,\ \alpha K_m^n \sin v;\ \beta l_m^n \sin v,\ \beta L_m^n \sin v;\ \gamma m_m^n,\ \gamma M_m^n$$

according to spherical functions. The distribution of the given values on randomly selected parallel circles does not make it impossible to apply the Neumann method[1] to the development according to these functions, but it is disadvantageous, since the series terminates at a large distance from the poles. In order to avoid this, and to apply that method in its simplist form, I would have had to employ the cartographic representation of the elements and determine their values at a number of new latitudes chosen correspondingly. In fact, I consider it necessary to carry out this study subsequently. In the present case, an important advantage would have been lost in this manner. The possibility is gained of observing the effect of the altered manner of treatment very precisely, because the calculations carried out by Neumayer and Petersen according to the simplified theory are /22
based on precisely the same empirical numerical values as those undertaken here according to the completed theory. Because of the considerable differences between observation and calculation, on the other hand, the comparison of results obtained in both cases would have suffered in the application of different parallel circles in one and another case in a manner not to be overlooked when the cartographical representation used as a basis had remained the same, as in this case.

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[1] Astronomische Nachrichten, Vol. 15, p. 313 and Mathematische Annalen, Vol. 14, p. 567 (reprint). I have given a short compilation of equations (without explanation) in my first article (Aus dem Archiv der Deutschen Seewart, 1889, no. 3). Seeliger provided a more detailed discription with tables in the session reports of the mathematical-physical class of the academy of Sciences in Munich, 1890, p. 499.

Therefore, I have employed the method of smallest squares for deriving the development in the customary manner. $f_{m,i}$ is any of the coefficients

$$\alpha_i k_{m,i} \sin v_i, \quad \alpha_i K_{m,i} \sin v_i, \quad \beta_i l_{m,i} \sin v_i, \quad \beta_i L_{m,i} \sin v_i, \quad \gamma_i m_{m,i}, \quad \gamma_i M_{m,i}$$

where the index i passing through the values 1, 2... 25 differentiates the values belonging to the different parallel circles

$$u_1 = 30^0, \; u_2 = 35^0, \; \ldots\ldots\ldots \; u_{25} = 150^0$$

and F_m designates the corresponding value under the coefficients B_m, C_m, D_m, E_m, j_m, k_m. The system of error equations is then expressed as

$$f_{m,i} = \sum_{n=m}^{n=m+\nu} F_m^n \cdot R_m^n (\cos v_i) \qquad i = 1, 2 \ldots 25$$

The resolution of this system also depends on the weight determination. It could be possible to introduce the equations refering to relatively unknown areas, e.g. to higher southern latitudes, with less weight then the others. It is, however, easily seen that this entails the danger of buying the gain in reduction of random error with a systematic distortion of the results in this manner. Moreover, the question must be considered from another aspect. The quantities for which the series development is sought are, when the flattening of the earth is neglected for reasons of simplicity, X sin u, Y sin u and Z, while the observed values, are those of X, Y, Z. Compensation would then be undertaken by the most precise representation of the latter, while this is carried out for the former in the assumption of the same weight for the error equations. Accordingly, the weight should only be introduced as constant in the development of Z, /23 while it should be set inversely proportional to the square of the sine of the width in that for X sin u and Y sin u. On the other

hand, it must be considered that the areas around the equator actually do not have such densely arranged, observed values in the calculation as the areas closer to the poles because of the decreasing length of the parallel circles with increasing latitude, of course, disregarding the exceptional polar areas. In order to compensate for this differing density, in other words, to permit an effect on the result on the basis of the individual zones corresponding to their area, it would apparently be necessary to multiply the weight of each equation with sin u in all three components. The combination of these different aspects would therefore lead to the setting of the weight with (1:sin u) in the development of X sin u and Y sin u, but in the case of Z to select the reverse sin u for this purpose.

When it is now considered, however, that in the relatively large number of excess equations the selection of weight has only a slight effect on the result, insofar as this stems from random errors, since in this case the ratio of the extreme values for weight is only that of 1:2, it appears justified to decide the question according to practical considerations. These, however, support giving all equations the same weight. The establishment of normal equations becomes somewhat easier then and, above all, the resolution is the same for all three components, while two systems of normal equations -- one for X sin u and Y sin u, the other for Z -- otherwise have to be prepared and solved. Finally, a justification based on fact can be supplied for this procedure. The present task does not finally deal with the compensation of the observed state considered erroneous according to a theoretically derived, enclosed equation, but rather the analytical representation of the state as it has been observed in an arbitrarily cut-off derivation, therefore only approximate. Therefore, as long as the remaining differences are not reduced below the amount of suspected uncertainty of observations, they are to be decreased by continuation of the derivation. The task then at least becomes approximately defined and the determination of weight is no longer important. This situation becomes most clear when the observations are considered as distributed in sufficient density

over the entire surface of the earth. In this case, where the individual coefficients are represented independently of one another by the known integrals, the task of compensation is relegated to /24 the background. It is apparently appropriate for the procedure to be applied then, when Z is treated in the same manner as X sin u and Y sin u or to return to the more precise representation γZ, as well as αX sin v and βY sin v in accordance with these considerations. I have introduced all equations with the same weight in the formation of the normal equations.

Concerning the limitation of the derivation, the final decision on this mater naturally depends on the permissible deviation between observed and calculated values. For the present, it appearedpractical to undertake an attempt with a derivation extended to the members of the sixth order, since the members of the first four orders are not sufficient without a doubt according to previous experience, on the one hand, and, on the other hand, the extent of the calculation, rapidly increasing with growing number of members, forces a limitation. (Proceeding only one order further would have been half a step, because the members of even and uneven orders are found independently of one another as a result of the symmetrical distribution of the 25 parallel circles employed.)

Formally, it must be required of the limitation for a derivation that it is not altered by a mere transformation of coordinates. It would follow from this, as the derived expression for the for the spherical function $P^n\,(\cos v_1 \cos v_2 + \sin v_1 \sin v_2 \cos(\lambda_1 - \lambda_2))$ shows that the series must conclude with those members having agreement for the indices m and n. Since in the method employed here as previously, first leading to the derivation of individual parallel circles, obtained independent of one another from the various multiples of the geographical length of the dependent portions of the function to be derived, there is no objection to defining the maximum value of m without consideration of the value of n with the exception of the understood condition that in each individual member m may not be larger than n.

I have expanded the derivation up to the values $m=4$, $n=6$ (in the case of $\alpha X \sin v$, then $n=7$). The remaining differences between the observed and the calculated values are distributed as will be later shown, almost equally to the members applied for m greater than 4 and that for n greater than 6, so that the two limits may be considered equal in a practical respect.

Since the calculation cannot be presented completely, as /25
already remarked, I dispense with it in the hope that I can provide the complete information later, even in an abstract, and now turn to the results.

From the normal equations, formed in the previously indicated manner, the coefficients of the series for $\alpha X \sin v$, $\beta Y \sin v$ and γZ result (under the common designation p_m^n, q_m^n for reasons of brevity), compiled in the two columns designated with IV and in the last column of Table VII (the significance of the numbers in rows I, II, III will be explained on page 38). These same numbers are found again in Table VIII, rounded off to whole numbers of the unit employed here and in somewhat altered, easily understandable arrangement. The fact that some of the numbers occurring here still contain decimal places is a result of the conditional equations (6). The values compiled in VIII are assumed as final and employed as a basis for all further calculations.

It is not necessary to explain that the small deviation introduced through the rounding off causes only a negligible increase in the error square sum as a result of compensation of the best values so that rounding off is thoroughly permissible; in contrast, it could be considered unnecessary, since further calculations have no noticeable advantage because of this. From this standpoint, the unaltered values given in Table VII would be justifiably preferred. The alteration chosen by me, however, has another purpose. The numbers found in VIII without decimal places are not to be considered approximation values, only rounded off one place more than those in VII, but rather they represent precise values, exactly defining a definite possible state of

the magnetic field of the earth, although only approximating the real condition. The numbers provided with two decimal places do not contribute to the definition of this state, as they result from the other coefficients on the basis of the conditional equations (6). The values indicated here are therefore to be considered as rounded off, and can be calculated more precisely with the aid of those equations (in the numerical calculation of X from the series for $\alpha X \sin v$, it is not practical near the pole, and impossible at the pole itself to form the sum

$$\frac{1}{\alpha \sin v}(B_0^0 R_0^0 + B_0^1 R_0^1 + B_0^2 R_0^2 + B_0^3 R_0^3 \ldots) \text{ oder } B_0^0 \frac{R_1^0}{\alpha \sin v} + B_0^1 \frac{R_1^1}{\alpha \sin v} + \ldots .$$

since this is converted into an indefinite expression for $v=0^{\circ}$ and for $v=180^{\circ}$. The conditions are the same in the calculation of Y. In order to avoid this negative situation, the series /26 is converted, taking into consideration the conditional equations existing between the coefficients (expressing the existence of a clearly defined horizontal force at each pole), possibly carried out in different manners. It is easiest to select the form

$$B_0^2 \frac{R_0^2 - \beta_0^2 R_0^0}{\alpha \sin v} + B_0^3 \frac{R_0^3 - \beta_0^3 R_0^1}{\alpha \sin v} + B_0^4 \frac{R_0^4 - \beta_0^4 R_0^0}{\alpha \sin v} + B_0^5 \frac{R_0^5 - \beta_0^5 R_0^1}{\alpha \sin v} + \ldots$$

$$\beta_0^{2n} = \alpha_0^{2n} : \alpha_0^0 = \alpha_0^{2n}, \quad \beta_0^{2n+1} = \alpha_0^{2n+1} : \alpha_0^1 = \alpha_0^{2n+1} : \sqrt{3}$$

because then the same function tables may be employed, no matter how far the derivation is carried. In consideration of this, I employed the first two coefficients (with n=m and n=m+1) as functions of the following in the case of m=0 and then to retain the same form, also in the remaining values of m. It might be preferred from another point of view to proceed in reverse and express the last coefficients of each series by the previous ones; however, the procedure is a question of subordinate significance.)

The same considerations, leading to the determination of weights of the equations (p. 22,23), make the derivation of medium or probable errors of the calculated coefficients (in VIII) from the differences present between observation and calculation appear

insignificant as long as these differences may be substantially reduced by a somewhat greater expansion of the series derivation. I prefer to provide the differences individually in order to permit clear judgement on the usefulness of the analytical representation gained, but then to undertake a mere estimation of the average error of the coefficients on the basis of the average uncertainty to be set for the observed values. Both data together show whether and which expansions of the series development are necessary in the present state of our empirical knowledge, when precision of the representation adapted to this is to be attained, and which further improvements may only be gained by new certain observations.

The following values of the coefficients k, K l, L m, M from 5 to 5 degrees for the entire range of $v=0^o$ to $v=180^o$ in the tables XI a, b, c are found from the numbers of table VIII. The numbers are indicated up to a tenth of the unit applied here so that later new calculations may be supported on precise differential values. The deviations of these calculated values /27 from those listed in VI a, b, c are found in the following tables XII a, b, c, rounded off to whole numbers so that they are more understandable. All these differences and those following are formed in such a manner that they represent the excess of the observed value over the calculated value. They therefore designate the portion of the observation results not yet reproduced by the analytical expression and are of the same type as these. Any improvement in the latter may not be directly considered as correction in the differences, and when these are represented completely or partially analytically, the results are to be added without alteration to the already determined expressions.

The values designated as observed for the coefficients k, K etc. are only indirect, in contrast to those derived from the spherical function series. Those numbers can be termed actually observed, defining the empirical facts employed as a basis, i.e. the values of X, Y, Z in the 1800 points of the earth's surface

described in the beginning (the fact that these are also already more or less the result of processing the direct measurement results is not important for the present report). A conclusive judgement can then be founded only on the comparison of the values determined at these 1800 points according to Dr. Neumayer's charts with those following from the representation in Table VIII. The tables in XIV a, b, c serve for this comparison, but in these only the 156 points already occuring in Table V -- from 10° to 10° in latitude, from 30° to 30° in longitude -- have been taken into consideration, sufficient for gaining a survey. Each of these difference tables is divided into three sections. The first, headed by Δ', contains those differences, occurring because of the limitation of the trigonometrical series derivations for the members of the first four orders, in other words, the excess of the observed values (in V) above those calculated according to the tables VI a, b, c. The differences produced by the limitation of the development according to spherical functions are found under the designation Δ'', resulting directly from the numbers in Tables XII a, b, c. The sum of those differences, i.e. the total difference between observation and calculation finally forms the third section Δ, of the tables.

In order to evaluate the numbers given here, it must be noted that the average intensity of the total force of the earth's magnetism is approximately equal to 50000 (with the extreme values of 26000 and 70000), measured in the same unit, while the average values of X, Y, Z with the exception of the signs are applied at approximately 30000, 6000, 4000. A direct comparison with the deviations obtained in earlier calculations of potential is not possible, because these can only be expanded to the spherical functions of the fourth order (at least insofar as they have been published). A comparison may be gained indirectly, however, with the aid of my provisional calculation mentioned in the beginning, also proceeding to the fourth order. In this case, the differences Δ were almost 1.8 times as large as the definitive calculation presented here, although only the component Δ'' was reduced in these. Specifically, the average differences calculated app- /28

roximately with the aid of Fechner's equation from the averages of their 156 absolute values

	in the case of X	Y	Z
after the first calculation	$\pm$ 762	610	1331
after the second calculation	$\pm$ 336	434	819.

The introduction of the spherical functions of the fifth and sixth order (in the case of $\alpha X \sin V$ those of the sixth and seventh orders) therefore produces a substantial improvement of the results. Of the calculations carried out earlier, the Erman-Petersen calculation is best suited for comparisons, because only in this case are the differences between observation and calculation given with respect to the components of force[1]. These differences, however, refer to other points than those chosen here and their number is also less (90 versus 156); however, this condition should have only a slight effect on the comparison of both sides of the results. It is more important that the 9 coefficients of the trigonometrical series employed by Erman and Petersen are only derived from the elements of the 9 points of the corresponding parallel circle subsequently taken into consideration in the check, so that the differences designated with Δ' disappear there and the entire differences appear relatively too small in the case of Erman-Petersen. According to the data on p. 24 of the abovementioned publication, the probable error of a calculated value now amounts in conventional units to /29

in the case of X	Y	Z
$\pm$ 21,41	15,98	29,10,

producing the average error in the unit employed here through

[1] A. Erman and H. Petersen, "Fundamentals of the Gaussian theory and the phenomena of the earth's magnetism in 1829, " Berlin 1874, p. 24.

multiplication with 34.941 : 0.6745

in the case of	X	Y	Z
	$\pm$ 1109	828	1507

These differences are 1.3 times as large on the average as those given previously, corresponding to my provisorial calculation and 1.9 times as large as the actual differences Δ'' to be compared with them. It cannot be determined with certainty even in an intensive study how far this result, favorable for the latter calculation, is based on the fact that the new observation material is better than the old material. Partially, the superiority of the new calculation is doubtlessly based on the fact that each component is represented independently in it (of course, taking into consideration the purely analytical, non-physical conditional equations combining X and Y), while all three components were found according to the old method, on the basis of certain physical hypotheses (p. 3) by a mutual compensation. Therefore, the number of the constants to be determined and employed as a basis for the representation is three times as great as here. Therefore, the new calculation must already produce slighter differences, when these are based only on observational errors; this must be the case to a higher degree, when the theoretical assumptions of the earlier calculation are not fulfilled (the hypothesis of the existance of a potential of exclusively internal forces). The resulting contradictions between the values of the three components not fitting into the assumed form must be included in the final differences, so that these cannot be reduced below a certain amount by any expansion of the series, no matter how far. In contrast, the differences in the independent calculation of individual components, as appearing in XIV a, b, c may be made randomly small. The superiority of the new method over the old must therefore become more emphasized as more members are taken into consideration in the derivation. Previous experience corresponds to this expectation and therefore supplies an indirect proof for the fact

that the physical prerequisites of the old theory could not be strictly fulfilled. While the addition of the spherical functions of the fifth and sixth order in present calculations resulted in a substantial reduction in differences, as was already noted (p. 28), the same expansion of the derivation in Dr. Neumayer's calculation /30 of potential (the only one expanded up to now beyond the fourth order) had no substantial use[1], although in this case, the members dependent on 5λ and 6λ were taken into consideration, reducing the differences Δ' in contrast to the present calculations.

Although the result of the new calculation may be designated as relatively favorable, it is still not satisfactory. The differences compiled in the tables XIV a, b, c are apparently still substantially greater than the possible errors of the observed values (i.e. taken from the charts), for which it is hardly possible to gain even a relatively certain estimation. Therefore, a continuation of the development could be considered. Some considerations also at least contradict a considerable expansion of the series and make it appear useful to proceed in steps, empirically determining the most practical limit for the derivation on the basis of successive results. The larger the number of considered series members chosen and therefore the unknowns to be considered, the greater is generally the influence of observation errors on the coefficients. A continuation of the derivation therefore causes better adaptation to the finite number of observation values available, but the entire representation becomes less certain, worsening in the intermediate points. This is to be anticipated in especially high degree in the case of the calculation basis employed here, containing two large areas completely free of observations, the two polar circles of 30° spherical radius. When the precision is too great in the representation of the values given for the zone from 60° to the north to 60° to the southern latitude, the derivation for the polar areas would be completely unuseable. In addition,

)1 Neumayer, "On magnetic surveying of the earth," lecture given at the meeting of German Scientists and doctors in Bremen, 1890.

there is the consideration that an almost complete representation of the observed values through an analytical expression is not even the most important task. Especially because this task can be solved theoretically with random approximation, as it is expressed here, while the lack of rules for the individual phenomena excludes rapid convergence of the series or a regulated correspondence of coefficients, the actual performance of the series derivation up to members of very high order is theoretically uninteresting and practically worthless. The main significance of the analytical representation is that it alone can supply a reliable basis for the physical theory of the phenomenon, that even the first question about the spatial origin of the force must remain unanswered, for example, without it. The analytical representation permits individual consideration of the particular solutions of the fundamental differential equations of the problem by separating the entire picture of the phenonemon without rules into a (infinite) sum of regular components. The result is that the conclusions to be drawn from the series derivation are dependent on the expansion of the series. Therefore, the number is not as important as the most precise possible determination of the series members to be calculated and the most important task must be presently considered an essentially precise determination of the first coefficients of the series. Of course, where each normal equation contains several unknown coefficients as a result of the distribution of the observed values so that these cannot be obtained independently of one another, both cannot be completely separated. Up to a certain unpredictable limit, therefore, only the expansion of the series derivation leads to a more precise determination of the individual coefficients, especially the first and most important. A completely independent calculation of each coefficient and the possibility of obtaining its value free of any uncertainty attributed solely to observation errors requires the knowledge of the functions to be represented (for the components of force) on the entire surface of the earth, therefore appearing desirable here once again.

/31

In the preceeding observations, only the purely analytical representation of the empirically provided distribution of force, clearly defined by the numbers of Table VIII have been treated. With the aid of the equations compiled in the first sections, a second equivalent representation is to be derived, determined in form according to physical considerations, serving as a basis for studying the causes of this distribution of force. For this purpose, the functions U and W are to be derived first from αX sin v and βY sin v. Their coefficients are found in Table IX, also containing those of (V:b), i.e. the arithemetic mean of U_0 and W_0. The circumstance that U and W are not in agreement expresses according to what has been said earlier that the horizontal forces in the earth's surface cannot be attributed completely to a potential. When an electrical current is assumed as cause for this, penetrating the earth's surface perpendicularly, the intensity in relation to the surface unit can be calculated, in other words the current density i with the aid of the equation (14) from the difference (W–U). An enclosed derivation results for the almost proportional quantity $\alpha\beta$bi, with coefficients to be found in Table X. The average amount of bi on the entire earth's surface results from this at about 110, i.e. 0.0011 $cm^{-\frac{1}{2}}\ g^{\frac{1}{2}}\ s^{-1}$. Since b, the polar radius of the earth, amounts to $6.356x10^{8}$ cm, the average result for i results with $1.7x10^{-12}$ $cm^{-\frac{1}{2}}\ g^{\frac{1}{2}}\ s^{-1}$ or approximately $1.7x10^{-11}$ Ampere: cm^2. (Of course, square agents are meant here, since the algebraic sum of the currents and the simple average of the intensity values disappear.) Therefore, on an area of 1 square kilometer there is an average current of 1/6 Ampere. The physical possibility of such a weak current cannot be argued; there is no reason from this side to doubt the actual difference of W and U. Another question to be studied is whether or not the uncertainty of the coefficients of U and W is so large that the difference of the two functions becomes illusory. /32

From the potential V of the main portion of the horizontal forces, chosen as large as possible, and from the vertical component V, the potentials V_i and V_a of the inner and outer agents result

according to the equations 15/18. The coefficients of both, or rather the proportional quantities (V_i: b) and (V_a: b) are given in table X. It can be seen that V_a isactually very small, as was predicted according to the earlier potential calculations; the average amounts to only approximately 1/40 of that of V_i. Of course even here, the certainty of results must still be examined. When I first dispense with this, I can express the result according to the calculations in the following manner.

The empirically determined distribution of the magnetic forces of the earth in the earth's surface, as they were represented in Dr. Neumayer's charts for the moment 1885,0, is based mainly on causes situated in the earth's core, but cannot be attributed exclusively to these. A small portion of the force, (about 1/40 of the entire amount) can be attributed to causes outside of the earth's surface; a further still somewhat greater portion (especially the horizontal component) points to electrical currents penetrating this surface.

The two representations of the magnetic state of the earth, /33
defined on the one hand by the numbers in VIII and those in X on the other hand, have this same value, as was already noted. One can be derived from the other, taking into consideration the earlier-mentioned side conditions (p. 10). It is therefore understood, but still deserves special emphasis, that the representation given in X deviates from the observed values only by the differences already present earlier (in the case of VIII), expressed in XII and XIV. It is therefore possible in principle to have the representation through V_i, V_a and i approach the actual, empirically determined state with random accuracy.

The currents parallel to the earth's surface can easily be calculated from the values found for V_i and V_a (compare equations 22), producing the forces expressed therein. I will dispense with providing the coefficients of the corresponding derivations, since they may be considered relatively uncertain. In the case of V_i,

the possibility that at least a noticeable portion is to be attributed to a direct magnetization of the earth's crust cannot be rejected; on the other hand, in the case of V_a the uncertainty of the values found, although not absolute, but still relatively large, is too considerable to provide sufficient value for the derivation of detailed conclusions. Therefore, I will limit myself to a summary, only characterizing the magnitude of the currents necessary for explaining the phenomena. In this case, I imagine in the manner described earlier (p. 14) that the entire flow is compressed through vertical shifting into two infinite, thin layers infinitely adjacent to the earth's surface on both sides and indicate the current, passing through a cross-section perpendicular to its direction of 1 cm in width. The average intensity expressed in this manner is rather precisely 1/2 AMP: cm for the internal and approximately 1/700 AMP: cm for the external current, when the meridianal component, rather slight in the case of V_i and hardly certain in the case of V_a, is disregarded. The direction of both goes from east to west.

It is now appropriate to list some further results of a similar, general significance, the moment and the axial direction of the earth magnet, determining the effect at a distance. These two concepts must be limited to the portion of force originating within the earth. The external agents expressed in V_a could be represented by a moment of defined direction and size only with specific assumptions on distribution, when this concept is understood in the customary manner. /34 In contrast, however, a vector corresponding to the moment could be indicated in the case of V_a in another interpretation, determining direction and strength of the magnetic force at the center of the earth. This data would probably not be especially interesting; therefore, I will not provide the appropriate results.

The value results for the magnetic moment (it could be termed that of the solid body of the earth) under the given limitation

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$$M = 8{,}3481 \cdot 10^{25}\, cm^{\frac{1}{2}}\, g^{\frac{1}{2}}\, s^{-1} \qquad (\log M = 25{,}92159)$$

and as axial direction those according to the point

$$\begin{aligned} & v = 168^{0}\,32',1 & \lambda = 111^{0}\,29',4 \\ \text{or}\quad & u = 168^{0}\,34',3 & \lambda = 111^{0}\,29',4. \end{aligned}$$

The axis therefore connects the point of $78^{o}34',3$ northern latitude and $68^{o}30',6$ western longitude from Greenwich with that situated at the $78^{o}34'3$ southern geographical latitude and $111^{o}39',4$ eastern longitude.

For comparison, I mention that the moment according to the calculation of Neumayer and Petersen amounts to $0.32237R^3$, i.e. $8.3324\mathrm{x}10^{25}$ $cm^{\frac{1}{2}}$ $g^{\frac{1}{2}}$ s^{-1}(with $R=6.370\mathrm{x}10^8$ cm) and that the axis is directed to the point $\phi=-78^{o}20'$, $\lambda=112^{o}43'$. The slight deviation of the numbers from those given previously shows clearly how much the causes of the magnetic force of the earth is situated in the earth's core. The question still remains to be studied on the certainty of all these results and which conditions are to be fulfilled to increase the certainty.

When the indicated values, considered as observed, were completely free of error, i.e. when they correspond in complete precision to the actual space of the earth's magnetism in a defined moment (even with elimination of exactly defined local and temporal disturbances), the differences (portions of which are given in Tables XIV) of the calculated and observed values would be a consequence of the early interruption of the series and it would be possible to reduce them through a continuation of the derivation. It would not be practical, however, even in this favorable case, to continue the derivation so far that the differences disappear; this would only be advantageous if it were known a priori that all members

of higher order in the exact representation (only obtained by integration over the entire surface of the earth), not determined in the calculation of individual values, are actually negligible. /35 If this is not known -- and in the present case the contrary can be concluded from a mere survey of the entire irregular and slowly decreasing coefficients -- the representation of the above-mentioned values would be improved, as was noted earlier, but those of the entire state would be negatively affected. This is the case to an even greater degree, when the given numbers are erroneous, so that the task assumes the character of a compensation calculation.

Therefore, as long as the observation basis cannot be expanded, a type of scanning procedure is recommended, providing an evaluation through successive expansion of the series derivations on which alterations are undertaken for the first, most important coefficients and the degree to which their terminal values may differ from the limit they approach in the further derivation. Now I have undertaken a provisorial representation, proceeding only to the fourth order, as was noted earlier; moreover, I have also carried out a greatly abbreviated derivation expanded only to the second order (in the case of $\alpha X \sin\ v$ of course to the third order). (Since the members of even and uneven order are calculated independently of one another, so that the two intermediate cases are processed for which the derivation goes to the third and the fifth order.) I shall not provide the results with the exception of some examples, but only give a summarizing result that those portions of the derivation dependent on a multiple greater than double the geographical length λ may only be assumed as raw first approximations, and, on the other hand, that the first coefficients are to be determined so precisely for m=0 and m=1, to a slighter degree also for m=2, that a continuation would not alter them very much. Careful examination of the differences given in Table XII leads to the same conclusion. A further circumstance is important for the evaluation of results. The two derivations apply to $\alpha X \sin\ v$ and $\beta Y \sin\ v$ are connected by a number of purely analytical conditional equations not proceeding for physical reasons, according to earlier explanations (p. 11). It is now shown that these equations generally introduce

a surprisingly strong constraint. I first determined the coefficients of both derivations independently and subsequently calculated the alterations caused by the introduction of the conditional equations and added these. The result of these alterations was very /36
considerable in almost all cases, as was already mentioned; they decrease with the continuing expansion of the series derivation and are smallest in the first members of the series. The main reason for the contradiction appearing in these differences lies in the irregular distribution of the magnetic force of the earth, only represented by slowly converging rows; the lack of observed values from the polar regions has a distorting effect especially here, and this is easily understandable when it is noted that those conditional equations refer to the arrangement of forces at the poles. The errors involved in the given values have only an indirect effect, insofar as they supply relatively large amounts in their irregular distribution, especially to the members of higher order numbers.

An example is the portion multiplied by $\cos\lambda$ of the series for $\alpha X \sin v$ and the related factor of $\sin\lambda$ in the series for $\beta Y \sin v$.

When the derivation is expanded up to R_1^3 in the case of X, that is R_1^2 in the case of Y, the coefficients defined without consideration of the conditional equations are

$$\begin{array}{rrr} -1943{,}9 & 198{,}9 & 732{,}9 \\ -1341{,}3 & 1273{,}8; & \end{array}$$

while the values actually to be chosen and satisfying the conditional equations are as follows:

— 1872,1	— 455,5	1128,5
— 1018,6	1051,2.	

When the derivation is carried two steps further, the numbers result as independently calculated:

— 1930,5	205,7	757,6	— 997,5	178,9
— 1290,8	1272,5	— 525,9	189,7	

and when compensated:

— 1913,2	270,8	817,1	— 790,2	308,7
— 1312,3	1249,6	— 644,1	116,8.	

When two further members are taken into consideration in each series, the corresponding coefficients are

— 1932,3	270,4	752,1	— 899,9	172,0	313,4	— 13,9
— 1279,8	1272,1	— 505,5	189,1	147,9	— 1,7	

and /37

— 1920,9	258,4	788,5	— 924,4	226,9	273,9	54,6
— 1276,9	1260,9	— 495,5	166,1	169,7	— 38,7.	

Consideration of these numbers demonstrates that only the first coefficients are somewhat certain and that a separate calculation of the individual coefficients is desirable. Such a procedure, however, requires the knowledge of the magnetic elements in the polar areas (although they are not undertaken through evaluation of surface integrals but according to the Neumann method).

When the corresponding parts of the series for (V:b) and $\alpha\beta bi$ are calculated from the previously reported partial developments of X and Y, similar differences must of course result in the progressive expansion of the series. The coefficients of $R_1^1 \cos\lambda$, $R_1^2 \cos\lambda$ .... in (V: b) in the three cases under consideration are

— 1019,7	1115,3				
— 1506,6	1252,4	— 591,8	137,0		
— 1220,4	1231,6	— 451,3	156,9	140,4	— 9,8

and those of $R_1^1 \sin\lambda$, $R_1^2 \sin\lambda$ ... in $\alpha\beta bi$

— 45,8				
— 30,4	— 52,1	— 26,0		
— 33,3	3,3	— 39,7	47,9	— 54,5.

The coefficients of the series for γZ limited by no conditional equation (with the exception of the one $j_0^0=0$) demonstrate generally small alterations in the continued derivation. The coefficients belonging to $R_1^1 \cos\lambda$, $R_1^2 \cos\lambda$ ... will be listed here as illustrations. In the three cases they are expressed as follows:

3083	— 3819				
2893	— 3813	1984	— 868		
2847	— 3841	1900	— 909	— 607	— 132.

Concerning the above-mentioned conditional equation $j_0^8=0$, this is fulfilled almost without any constraint, in contrast to the previously mentioned equations. When the coefficients j_0^0, j_0^2, j_0^4, j_0^6 are determined without consideration of that equation the results are 5.9 -- 708.5 -- 1265.2 -- 32.7, while the results under consideration of the conditions are 0 -- 701.4 -- 1272.6 -- 40.0. The previous considerations give the impression that the relatively small differences for concluding the existance of a current penetrating the earth's surface and of a potential /38
of external agents must be termed illusion. This impression is intensified by further considerations. Table VII contains four columns each in the case of X and Y, but only the last one has been mentioned up to now. The numbers in the three preceeding columns designate the coefficients of the series under the following assumptions:

I. $i=0$, $V_a=0$, i.e. the magnetic force of the earth is based exclusively on the effect of inner causes, as was assumed by the Gaussian theory. Under this assumption the series for $\alpha X \sin v$ and $\beta Y \sin v$ are calculated from those for γZ.

II. The series are derived without consideration of the conditional equations (some of the numbers listed here were already mentioned (p. 36)).

III. $i=0$, i.e. the series has been calculated under the assumption that the entire force exhibits a potential partially based on external causes.

It can be seen that the result of the calculation carried out without any hypothesis, listed in column IV, does not follow any better from II than does III, while I deviates somewhat more. In this case, it must be considered that I is supported on the values of γZ, assumed to be unaltered; in a common compensation, of X, Y, Z (as is undertaken in the Gaussian procedure), instead of I a value system situated between I and III was the goal.

In order to illuminate the matter from still another side, I have calculated the values of $k_1 K_1 l_1 L_1$, although only for the portion dependent on cosλ and sinλ and have formed their differences to the observed values. These are found in Tables XIII a and b. It should not be forgotten in comparison that the error square sum is not formed from these values themselves, but from their product with sin u (or strictly with αsin v in the case of X, βsin v in the case of Y), so that larger differences in higher latitudes have less effect than such near the equator. In fact, it can now be seen that the differences in the case of IV more closely approximate those in the case of II than the other two, of course representing the error minimum. This supports the fact that the representation of the actual distribution of force exhibits clear progress in spite of all uncertainty of individual components given by their totality, when the hypotheses that i and V_a disappear are dispensed with.

I have not yet taken into consideration that uncertainty /39 stemming from the errors in observed values. The preceeding considerations are hardly affected by this, since the effect on the four parallel calculations cannot differ very much. The amount and direction of this effect on the final results will now be examined. Of course, it is presently impossible to carry out precise calculations and these will not be possible for a long time, because they assume the knowledge of extremely alternating uncertainty for the individual evaluated observations and surveys as well as those attached to the assumed secular variations, and while it is still difficult for broad areas to gain any useable values of the magnetic elements, it is much more difficult to form a judgement on their approximate precision. Even a raw estimation is not possible in this case in a direct manner, since it would have to return to the original material processed in the charts. A conclusion about the possible amount of average errors of the observed values can be drawn indirectly, however, from a consideration of the differences between the observed and calculated values.

The slow reduction in coefficients of the series development with increasing order number n removes any doubt that the remaining differences are attributed to a great part to calculation of the insufficient expansion of the series and the grouping of these differences still far removed from irregular arrangement leads to the same conclusion. When it is now observed how substantially the average deviation was reduced according to earlier data (p. 28) by the addition of the members of the fifth and sixth order, it can be assumed without a doubt that the average amount of deviations attributable to the inaccuracy of the observed values is at most half as great as the entire amount of these. Therefore, I will assume in round numbers that the values of the components of force have an average error of $\pm$ 200 in the case of X and Y and of 400 in the case of Z for each of the 1800 points utilized and these numbers can probably be considered an upper limit. Average errors of $\pm 20'$ and $\pm 0.002\ cm^{-\frac{1}{2}}\ g^{\frac{1}{2}}\ s^{-1}$ could be considered approximately equal in value in the declination and horizontal intensity and errors on the order of magnitude of 1° in inclination. Of course, all unknown effects of temporal or local disturbances, the inaccuracy of the observation itself and the uncertainty of reduction to the normal epoch, finally also those in the drawing of the isomagnetic lines and in the removal of values for certain points between the poles from the charts are contained in these numbers. /40

Since the extreme differences existing between the individual areas of the earth's surface with respect to the reliability of our knowledge on the magnetic state is completely neglected in this procedure, it can only be designated a summarizing estimation; however, substantial distortion of the conclusions to be drawn from this must not be anticipated, since the only consideration is the size of expected average uncertainty.

The coefficients $k_m, K_m \ldots$ of the trigonometrical series are derived from 36 individual values each; therefore, the average error results from the above-applied amounts by adding the factor 1/6 or 1/6 $\sqrt{2}$, according to whether the index m is equal to 0 or not. When ε is the absolute amount of error, then

a für $m = 0$ b bei X c und $Y: \epsilon = 33$ b bei $Z: \epsilon = 67$
für $m = 1, 2, 3, 4$ bei X und $Y: \epsilon = 47$ bei $Z: \epsilon = 93$

Key: a. For b. In the Case of c. And

(A justifiable further rounding off of these numbers will not be undertaken in order to prevent distortion of the relationships of various values.). The normal equations for determining the coefficients B, C, D, E j, k from the values of k_m, K_m ... also lead to the calculation of the average error of the former in a known manner. I again provide only some results, sufficient for characterization. In this case, I would like to note that I have assumed the values given for ϵ as average error of the quantities $\alpha k \sin v$, $\alpha K \sin v$... in order to substantially shorten the calculation for X and Y, although they belong to k, K... The errors of B, C, D, E resulting in this manner, a few of which will now be listed, are therefore somewhat too large, but this is not a problem since the only consideration is the calculation of an upper limit for this error.

In the case of m=0, the following anticipated average errors of the coefficients B^n_o, D^n_o, J^n_o result

$n =$		0	1	2	3	4	5	6	7
a bei X	$\pm$	(90)	(44)	15	13	19	13	17	14
bei Y	$\pm$	(90)	(24)	15	11	19	9	17	.
bei Z	$\pm$		52	51	60	45	55	36.	

Key: a. In the Case of

The values related to B_0^0, B_0^1, D_0^0, D_0^1 are enclosed in parentheses, while these coefficients are completely determined by the others /41 (p. 25) and therefore have no independent effect on the values of the function presented. In order to characterize the effect of the greater or smaller expansion of the series development on the average error of the coefficients, insofar as these stem from errors in observation material, a typical example may be sufficient. According to whether the calculation of the coefficients j_0^2, j_0^4, j_0^6 is interrupted with the first, the second or, as was undertaken in the present report, with the third, the average errors are

$$0{,}24\,\varepsilon; \qquad 0{,}37\,\varepsilon,\quad 0{,}37\,\varepsilon; \qquad 0{,}77\,\varepsilon,\quad 0{,}67\,\varepsilon,\quad 0{,}54\,\varepsilon.$$

These numbers demonstrate that an uncritical continuation of the derivation without expansion of the the empirical basis would not lead to an improvement, but a worsening of the results.

For m=1, which I will describe in detail later, since the results for m=2, 3, 4 are on the same order of magnitude, the following amounts of error result

	$n =$	1	2	3	4	5	6	7
[a] bei X	$\pm$	8	8	14	10	19	13	21
bei Y	$\pm$	8	8	10	11	12	14	
bei Z	$\pm$	13	14	13	17	16	21.	

Key: a. In The Case of

These error amounts depend of course on the formation of the normal equations; it would be decreased still more if the observations were distributed evenly over the entire surface of the earth.

The average errors given in the preceding, moreover representing upper limit values, are slight and hardly need be considered especially in the first coefficients of the series. The circumstance is a result of the fact that the derivation is based on the elements of an extremely large number of points. The analytical representation of the components of force can therefore be designated as very precise and reliable with the exception of the systematic error caused by series interruption.

The series for V_i, V_a and i are much less favorable. The anticipated average errors are not only relatively larger in this case, but they achieve the maximum precisely in the first and most /42 important coefficients. I will limit myself here to the presentation of an example; specifically, I chose those coefficients of the above-mentioned functions for this purpose, resulting from the previously indicated coefficients for X, Y, Z. These are the factors connected to R_0^1, R_0^2... and those with R_1^1, R_1^2.... I will compile them in an arrangement similar to that of the previous examples:

$m = 0$	$n =$	1	2	3	4	5	6
a m. F. bei $V_i : b$	±	31	15	12	7	6	3
m. F. bei $V_a : b$	±	41	16	14	7	7	3
m. F. bei $\alpha \beta b i$	±	14	7	18	7	16	
$m = 1$	$n =$	1	2	3	4	5	6
m. F. bei $V_i : b$	±	53	27	33	19	23	16
m. F. bei $V_a : b$	±	75	30	35	20	23	16
m. F. bei $\alpha \beta b i$	±	38	15	22	11	15	

Key: a. Average Error in the Case of

At least in the case of m=1 (and it is similar in the case of m=2, 3, 4) these error amounts, however upper limit values, often exceed the values of the coefficients of V_a and i. Especially these functions, interesting in a physical respect, therefore become so uncertain that it appears doubt ful whether they are perhaps only a product of observational errors. Some early experience,

however, supports the fact that this is not the case (p. 29, 38) and the individual derivation coefficients may indeed be very uncertain while the function presented assumes values of much less uncertainty at some positions. This is caused by the fact that the coefficients are dependent on one another, since they partially depend on the same variables (coefficients of the series for X, Y, Z).

The reality of V_a and i are supported by the circumstance that at least a portion of these functions noticeably exceeds the error limits, specifically those belonging to the index m=0 and therefore independent of the geographical longitude, so that it may not be considered a result of mere internal contradictions of the data provided. A more precise consideration intensifies this impression. The component free of λ in $\alpha\beta$ bi follows from the portions of the development of $\beta Y \sin v$ also only dependent on v with coefficients designated on the individual parallel circles by l_0. The condition that in the derivation according to λ complete circles with evenly distributed observation values are present (in contrast to the derivations according to v, supported here /43
only on the arc from $v=30^\circ$ to $v=150^\circ$) has the effect that the coefficients of the trigonometrical series may deviate from the true values almost solely as a result of the errors in empirical data. The effect of these random errors on l_0 was recently found (p.10) at an estimated ±33. When this is now compared with the values of l_0 given in Table VIb, growing even in the relatively well studied medium latitudes of the northern hemosphere up to 150, on the southern hemosphere even up to the double amount, it can be seen furthermore that these numbers exhibit a very clearly marked course, so it appears thoroughly impossible to attribute these to errors in observational material. This would only be possible when large systematic errors could be distributed over large areas (entire continents or oceans), a rather improbable assumption.

When the existence of vertical currents i has been proven with a great probability by means of the considerable values of

i_0, there is no reason to assume that the coefficients found in the derivation of i with the indices m=1, 2, 3, 4 are actually 0 and that the values found for this factor only stem from observational errors. These coefficients, however, have been determined with so much uncertainty according to the above discussion that an evaluation of the series found for i could only supply values of a rather doubtful nature. Therefore, I will not provide any information on the calculations I carried out in this direction.

If there are currents connecting the space inside the earth's surface with that outside, a potential of external forces in the earth's surface is necessarily proven. Independently, at least the strong value of the coefficient of R_0^1, and probably that of R_1^1, not attributed to faulty observation, also supports the actual existence of V_a.

Let us survey the essential results once again in context. It can first be stated that the initial task, that of the purely analytical representation of the observed phenomena, has been solved in an almost sufficient manner in the expansion of the series applied here. The remaining differences demonstrate that the addition of the members, depending on a multiple of 5 and at most also those depending on a multiple of 6 of the geographical lengths λ and those of the seventh and eighth order in the spherical functions of the polar distance v would certainly be sufficient to provide /44
a representation of the observed values with such precision as is desireable with respect to the degree of its inherent certainty. On the other hand, it is certain that the outermost limit is achieved in such an expansion of the series, and perhaps already exceeded, beyond which the increasing random error of the calculated coefficients cancel the gain in more precise representation and reverse it. This problem, however, would be somewhat reduced when the already very large number of 1800 points, from which the elements were utilized, is increased or when the derivation is supported only on the graphic representation of the elements; however, a considerable reduction in the resulting average error cannot be expected, because the largest portion of the earth's surface has

not been sufficiently surveyed so that the errors in adjacent points are considered mutually independent and may be treated as random errors.

Accordingly, when the total representation can be designated essentially as satisfactory, only the first coefficient of the series can be considered reliable. As a result, the series for V_i, V_a and i, derived from these coefficients, expressing precisely the physically interesting results, are already less certain when considered absolutely, because each of their coefficients is based on several coefficients of the original series. Moreover, since V_a and i are relatively small, the possible errors in this case are extremely large when considered relatively, so large that the actual existence of the physical conditions expressed by them is almost placed in doubt. And although both a potential of external agents and a component of the horizontal force not attributable to a potential may be considered proven, it must be admitted that nothing reliable can be determined on the precise shape of both with the present basis. Especially detailed knowledge of this shape, i.e. the distribution of the functional values of V_a and i over the earth's surface is manditory, when the question now arising about the nature of these phenenomena is to be treated. For example, how important would it be, only to note one possibility, when an analogy with the configuration of water and land or a connection to climactic areas was demonstrated in the arrangement of the positive and negative values of i. After what has just been said, it needs no justification to dispense with speculations on such /45 possible relationships, as well as any physical explanation of the phenomena, although several reasons could be found for this. I will only the note the conclusions, resulting, on the one hand, from the slow convergence of the series and, on the other hand, from the circumstance that the first members from which the magnetic moment originates dominates all others considerably; furthermore, I wish to mention at least the mutual effect of the inner and outer agents on one another. To treat this subject in more detail or some others is not only forbidden by consideration of the above-

described conditions; it would also extend beyond the framework of the present report, only aiming at the most exact possible analytical representation of the observed facts.

The utilization of material available from the areas north of 60 degrees in latitude and determination of coefficients in a modified manner already discussed (p. 21) would probably make the results appear somewhat more favorable. A substantial advance, however, with respect to the certainty of results can only be attained, as can be seen from the discussion above, when the calculation is based on complete knowledge about the distribution of the magnetic forces of the earth in the earth's surface. Of course, this requires, above all, gaining new reliable material from the southern polar region, but also undertaking new observations in numerous, scattered, accessible areas with special consideration of determining the secular variation. Too much precision in detail and a very detailed knowledge of the distribution of force in smaller areas, although this may be important for other, specific tasks, may be termed less significant for present purposes, insofar as it cannot compensate for the lack of observations at other points.

The demand results for future research from the results and considerations above that the consideration of the magnetic phenomena of the earth should be placed in the foreground as a unified phenomenon to a much greater degree than has been done in the past. The numerous individual studies of special tasks provided for research from a multitude of aspects should of course not be underestimated or neglected; however, although these may be important, they only obtain a true value through the classification in the relationship of the phenomena and this research should not be pushed in the background because of that. The most pressing task in the near future is to obtain a unified picture of the entire state of the magnetic system of the earth in its various relationships on the path of observation, first only in general characteristics and then later more detailed, and to follow its constant alteration. A planned, organized procedure is required for this purpose; this becomes even more necessary with less means for

research to solve these tasks.

darf noch viel weniger um jener willen zurücktreten. Als die dringendste Aufgabe der nächsten Zeit muss es daher bezeichnet werden, ein wenn auch zunächst nur in grossen Zügen gehaltenes, erst allmählich weiter auszuführendes, einheitliches Bild von dem Gesamtzustande des erdmagnetischen Systems in seinen verschiedenen Beziehungen auf dem Wege der Beobachtung zu gewinnen und seine stetige Aenderung zu verfolgen. Dazu bedarf es eines planmässigen, organisierten Vorgehens; es bedarf dessen um so mehr, je geringer die äusseren Mittel sind, die der Forschung zur Lösung ihrer Aufgaben zu Gebote stehen.

Ia. Tabelle der Koeffizienten ($\pi_m^n : \rho_m^n$).

m \ n	0	1	2	3	4	5	6
0	1,000000	1,000000	0,997765	0,997323	0,997133	0,997027	0,996960
1		0,993326	0,996663	0,996881	0,996896	0,996879	0,996859
2			0,993326	0,995551	0,996183	0,996436	0,996557
3				0,993326	0,994994	0,995697	0,996053
4					0,993326	0,994660	0,995347
5						0,993326	0,994438
6							0,993326

Ib. Tabelle der Koeffizienten ($\varkappa_m^n : \gamma_m^n$).

m \ n	0	1	2	3	4	5	6
0	0,995547	0,995991	0,996181	0,996286	0,996355	0,996400	0,996436
1		0,995325	0,995864	0,996103	0,996234	0,996316	0,996372
2			0,994913	0,995546	0,995870	0,996060	0,996181
3				0,994622	0,995264	0,995633	0,995864
4					0,994416	0,995035	0,995420
5						0,994266	0,994850
6							0,994151

Ic. Tabelle der Koeffizienten δ_m^n.

m \ n	0	1	2	3	4	5	6
0	1,004473	0,334227	0,200639	0,143325	0,111479	0,091211	0,077179
1		0,335125	0,200760	0,143368	0,111498	0,091222	0,077186
2			0,201151	0,143496	0,111556	0,091254	0,077205
3				0,143710	0,111653	0,091305	0,077236
4					0,111789	0,091378	0,077280
5						0,091473	0,077336
6							0,077405

Id. Tabelle der Koeffizienten c_m^n.

m \ n	0	1	2	3	4	5	6
0	0,000000	0,334227	0,400381	0,428825	0,444637	0,454701	0,461669
1		0,332885	0,400193	0,428763	0,444609	0,454686	0,461660
2			0,399617	0,428573	0,444522	0,454643	0,461632
3				0,428252	0,444378	0,454561	0,461586
4					0,444174	0,454452	0,461520
5						0,454311	0,461436
6							0,461332

II. Tabelle der Faktoren α, β, γ, $\alpha \sin v$, $\beta \sin v$.

u	v	α	γ	$\alpha \sin v$	$\beta \sin v$
0°	0° 0′ 0″	1,003354	1,000000	0,000000	0,000000
5°	5° 1′ 0″	1,003328	0,999974	0,087737	0,087739
10°	10° 1′ 58″	1,003252	0,999899	0,174778	0,174796
15°	15° 2′ 53″	1,003128	0,999775	0,260441	0,260500
20°	20° 3′ 42″	1,002960	0,999607	0,344047	0,344182
25°	25° 4′ 25″	1,002752	0,999401	0,424949	0,425204
30°	30° 4′ 59″	1,002512	0,999161	0,502514	0,502936
35°	35° 5′ 25″	1,002247	0,998897	0,576158	0,576794
40°	40° 5′ 40″	1,001964	0,998615	0,645314	0,646210
45°	45° 5′ 45″	1,001673	0,998324	0,709473	0,710664
50°	50° 5′ 40″	1,001382	0,998034	0,768163	0,769676
55°	55° 5′ 24″	1,001100	0,997753	0,820954	0,822802
60°	60° 4′ 59″	1,000835	0,997490	0,867473	0,869656
65°	65° 4′ 24″	1,000597	0,997252	0,907389	0,909890
70°	70° 3′ 42″	1,000391	0,997047	0,940427	0,943213
75°	75° 2′ 32″	1,000224	0,996880	0,966357	0,969382
80°	80° 1′ 58″	1,000101	0,996757	0,985006	0,988210
85°	85° 1′ 0″	1,000025	0,996682	0,996245	0,999561
90°	90° 0′ 0″	1,000000	0,996657	1,000000	1,003354

$\beta = 1,003354$

III. Tabelle der Faktoren r_m^n.

m \ n	0	1	2	3	4	5	6	7
0	1,000000	1,732051	3,354102	6,614378	13,12500	26,11842	52,05515	103,8444
1		1,732051	3,872983	8,100926	16,60196	33,71874	68,15619	137,3732
2			1,936492	5,123475	11,73936	25,48897	53,88229	112,1647
3				2,091650	6,274950	15,60874	35,92146	79,31244
4					2,218530	7,358031	19,67500	47,83720
5						2,326814	8,389446	23,91360
6							2,421825	9,379686
7								2,506827

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IV. Tabelle der Funktionen $K_n^m(\cos v)$.

$(\operatorname{tg} v = \operatorname{tg} u \sqrt{1+\varepsilon^2} = 1{,}003354 \operatorname{tg} u.)$

u	K_0^1	K_1^1	K_0^2	K_1^2	K_2^2
0°	1,732051	0.000000	2,236068	0,000000	0,000000
5°	1,725416	0.151460	2,210420	0,337378	0,014808
10°	1,705565	0.301743	2,134272	0,664401	0,058772
15°	1,672656	0.449691	2,009977	0,971058	0,130534
20°	1,626957	0.594147	1,841390	1,247943	0,227868
25°	1,568830	0.734013	1,633700	1,480633	0,347777
30°	1,498743	0,808199	1,393329	1,679851	0,480556
35°	1,417246	0,995698	1,127634	1,821785	0,639955
40°	1,324991	1,115526	0,844788	1,908171	0,803256
45°	1,222695	1,226792	0,553407	1,936481	0,971485
50°	1,111152	1,328662	0,262357	1,905954	1,139523
55°	0,991234	1,420372	— 0,019517	1,817618	1,302262
60°	0,803850	1,501254	— 0,285716	1,674237	1,454798
65°	0,729987	1,570707	— 0,522256	1,480248	1,592519
70°	0,590644	1,628232	— 0,727996	1,241557	1,711303
75°	0,446893	1,673406	— 0,894749	0,965448	1,807578
80°	0,299792	1,705909	— 1,017551	0,660237	1,878478
85°	0,150456	1,725504	— 1,092725	0,335159	1,921880
90°	0,000000	1,732051	— 1,118034	0,000000	1,936492

u	K_0^3	K_1^3	K_2^3	K_3^3	K_0^4
0°	2,645751	0,000000	0,000000	0,000000	3,00000
5°	2,585232	0,561295	0,039028	0,001399	2,88607
10°	2,407619	1,056188	0,153118	0,011059	2,55684
15°	2,124456	1,540816	0,333517	0,036606	2,04853
20°	1,754126	1,896107	0,566301	0,084429	1,41668
25°	1,320482	2,129881	0,833423	0,159191	0,72945
30°	0,851320	2,228245	1,113906	0,263430	0,05973
35°	0,376303	2,186571	1,385426	0,397365	— 0,52367
40°	— 0,074886	2,009743	1,625756	0,558788	— 0,96372
45°	— 0,474734	1,711741	1,814440	0,743224	— 1,22185
50°	— 0,799634	1,314642	1,934127	0,944170	— 1,28191
55°	— 1,031448	0,847103	1,971800	1,153488	— 1,15168
60°	— 1,158747	0,342267	1,919682	1,361976	— 0,86129
65°	— 1,177442	— 0,164361	1,775775	1,559864	— 0,45919
70°	— 1,091044	— 0,637506	1,543979	1,737625	— 0,00574
75°	— 0,910349	— 1,044301	1,233924	1,886301	0,43424
80°	— 0,652611	— 1,356706	0,860229	1,998364	0,79975
85°	— 0,340403	— 1,553165	0,441697	2,068020	1,04086
90°	0,000000	— 1,620185	0,000000	2,091650	1,12500

u	K_1^4	K_2^4	K_3^4	K_4^4	K_0^5
0°	0,00000	0,00000	0,00000	0,00000	3,31663
5°	0,81534	0,07626	0,00418	0,00013	3,12860
10°	1,54101	0,29457	0,03267	0,00204	2,59577
15°	2,09801	0,62493	0,10005	0,01008	1,80621
20°	2,42735	1,02149	0,23792	0,03072	0,88896
25°	2,49703	1,42847	0,43257	0,07155	— 0,00945
30°	2,30552	1,78712	0,68384	0,14006	— 0,75090
35°	1,88170	2,04323	0,97543	0,24229	— 1,23007
40°	1,28116	2,15398	1,28239	0,38172	— 1,39198
45°	0,57904	2,09348	1,57398	0,55835	— 1,24033
50°	— 0,13904	1,85615	1,81712	0,76821	— 0,83460
55°	— 0,78738	1,45779	1,98039	1,00330	— 0,27718
60°	— 1,29057	0,93386	2,03783	1,25210	0,30724
65°	— 1,59230	0,33567	1,97228	1,50039	0,79568
70°	— 1,66200	— 0,27565	1,77764	1,73256	1,09026
75°	— 1,49814	— 0,83593	1,46007	1,93298	1,13591
80°	— 1,12815	— 1,28565	1,03766	2,08760	0,92994
85°	— 0,60489	— 1,57648	0,53893	2,18518	0,52130
90°	0,00000	— 1,67705	0,00000	2,21853	0,00000

u	K_1^5	K_2^5	K_3^5	K_4^5	K_5^5
0°	0,00000	0,00000	0,00000	0,00000	0,00000
5°	1,09337	0,12796	0,00920	0,00043	0,00001
10°	2,00549	0,48472	0,07085	0,00667	0,00037
15°	2,58796	0,99431	0,22440	0,03229	0,00274
20°	2,75174	1,54670	0,48590	0,09570	0,01105
25°	2,48279	2,01952	0,84261	0,21496	0,03180
30°	1,84353	2,30204	1,25347	0,40194	0,07363
35°	0,96018	2,31720	1,65588	0,65753	0,14608
40°	— 0,00120	2,03711	1,97691	0,96848	0,25784
45°	— 0,86620	1,48936	2,14759	1,30725	0,41478
50°	— 1,48401	0,75264	2,11685	1,63452	0,61806
55°	— 1,75469	— 0,05708	1,86276	1,90433	0,86292
60°	— 1,64649	— 0,80784	1,39887	2,07116	1,13823
65°	— 1,20010	— 1,27557	0,77428	2,09727	1,42703
70°	— 0,51930	— 1,66718	0,06711	1,95952	1,70821
75°	0,24987	— 1,63757	— 0,62696	1,65412	1,95869
80°	0,94795	— 1,29832	— 1,21020	1,19840	2,15644
85°	1,43252	— 0,71589	— 1,59826	0,62955	2,28317
90°	1,60565	0,00000	— 1,73431	0,00000	2,32681

u	K_0^6	K_1^6	K_2^6	K_3^6	K_4^6	K_5^6	K_6^6
0°	3,6056	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
5°	3,3210	1,3901	0,1952	0,0172	0,0010	0,0000	0,0000
10°	2,5336	2,4581	0,7222	0,1303	0,0159	0,0013	0,0001
15°	1,4247	2,9637	1,4214	0,4006	0,0752	0,0096	0,0007
20°	0,2454	2,8089	2,0767	0,8303	0,2156	0,0374	0,0040
25°	— 0,7476	2,0605	2,4761	1,3562	0,4629	0,1039	0,0140
30°	— 1,3548	0,9299	2,4709	1,8635	0,8171	0,2297	0,0384
35°	— 1,4815	— 0,2847	2,0188	2,2157	1,2433	0,4310	0,0874
40°	— 1,1571	— 1,2769	1,1971	2,2939	1,6733	0,7112	0,1729
45°	— 0,5213	— 1,8123	0,1843	2,0327	2,0174	1,0557	0,3058
50°	0,2168	1,7860	— 0,7865	1,4431	2,1845	1,4296	0,4935
55°	0,8373	1,2462	— 1,4884	0,6211	2,1053	1,7806	0,7365
60°	1,1680	— 0,3754	— 1,7609	— 0,2798	1,7527	2,0468	1,0268
65°	1,1283	0,5623	— 1,5524	— 1,0737	1,1539	2,1685	1,3469
70°	0,7467	1,2961	— 0,9335	— 1,5920	0,3899	2,1003	1,6714
75°	0,1492	1,6213	— 0,0793	— 1,7231	— 0,4172	1,8221	1,9696
80°	— 0,4802	1,4544	0,7767	— 1,4421	— 1,1284	1,3458	2,2106
85°	— 0,9522	0,8535	1,4034	— 0,8181	— 1,6155	0,7151	2,3674
90°	— 1,1267	0,0000	1,6328	0,0000	— 1,7886	0,0000	2,4218

u	K_0^7	K_1^7	K_2^7	K_3^7	K_4^7	K_5^7	K_6^7	K_7^7
0°	3,8730	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
5°	3,4682	1,7009	0,2788	0,0291	0,0021	0,0001	0,0000	0,0000
10°	2,3787	2,8789	1,0031	0,2154	0,0321	0,0034	0,0003	0,0000
15°	0,9368	3,1881	1,8783	0,6389	0,1473	0,0241	0,0028	0,0002
20°	— 0,4300	2,5741	2,5376	1,2558	0,4053	0,0915	0,0144	0,0014
25°	— 1,3415	1,2859	2,6799	1,9038	0,8239	0,2430	0,0492	0,0062
30°	— 1,5870	— 0,2165	2,1839	2,3576	1,3533	0,5084	0,1287	0,0199
35°	— 1,1871	— 1,4269	1,1568	2,4143	1,8752	0,8897	0,2770	0,0520
40°	— 0,3725	— 1,9676	— 0,0997	1,9779	2,2312	1,3469	0,5121	0,1152
45°	0,5073	— 1,7148	— 1,1958	1,1079	2,2735	1,7964	0,8360	0,2242
50°	1,1128	— 0,8295	— 1,7914	0,0146	1,9207	2,1256	1,2261	0,3918
55°	1,2333	0,3123	— 1,7169	— 1,0023	1,1975	2,2224	1,6325	0,6252
60°	0,8546	1,2644	— 1,0327	— 1,6502	0,2420	2,0100	1,9835	0,9213
65°	0,1522	1,6744	— 0,0074	— 1,7420	— 0,7245	1,4769	2,1986	1,2644
70°	— 0,5833	1,4087	0,9791	— 1,2630	— 1,4582	0,6911	2,2074	1,6264
75°	— 1,0606	0,5940	1,5700	— 0,3801	— 1,7654	— 0,2084	1,9682	1,9697
80°	— 1,0966	— 0,4377	1,5585	0,6100	— 1,5642	— 1,0409	1,4819	2,2537
85°	— 0,6868	— 1,2790	0,9587	1,3765	— 0,9135	— 1,6279	0,7965	2,4412
90°	0,0000	— 1,6010	0,0000	1,6639	0,0000	— 1,8395	0,0000	2,5068

V. Beobachtete Werte der Komponenten der erdmagnetischen Kraft.[1])

X.

i	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	14031	16498	16367	15199	15866	17268	17314	14143	8009	4384	5639	9401
40°	17808	20605	21590	21601	22142	23378	22453	20155	16180	12227	11209	13092
50°	21992	25477	27628	28369	28324	27886	25078	24060	22879	20394	17332	17281
60°	25943	29508	33290	34362	33284	30799	28075	27594	28704	28069	24036	21042
70°	28462	32538	36080	37810	36100	33352	31167	30963	32962	32964	29129	25703
80°	29867	32901	35288	38347	37887	35713	34174	33820	35521	35557	30780	28447
90°	28143	30051	32950	36590	38573	37283	36414	35091	35306	34677	30482	28281
100°	24412	25548	28857	33358	36832	36714	36192	34798	33657	32702	29035	26297
110°	20900	21409	24251	28571	32996	34601	34220	32992	33559	29928	27395	24432
120°	19054	18126	19825	22852	20774	28866	29914	29916	28890	27553	26497	23361
130°	18429	15881	16252	17906	20322	22095	24561	26169	26174	26688	26697	23510
140°	18730	14685	13219	13014	14186	15610	19071	21646	25941	24851	27071	24443
150°	18966	14275	10831	7735	7143	8867	12950	16558	19002	21945	25822	24340

Y.

i	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	— 5129	— 240	3794	3037	— 656	— 1056	3865	7000	6140	— 636	— 7218	— 8539
40°	— 5643	— 1008	3227	2940	— 1600	— 1566	4739	7906	7176	748	— 6780	— 8665
50°	— 6133	— 1931	2255	2940	— 1773	— 893	5041	6975	7250	2084	— 5517	— 8366
60°	— 6725	— 2914	823	2604	— 1017	— 197	5187	5815	6585	2917	— 3771	— 8525
70°	— 7680	— 3803	— 210	1937	105	1378	5216	4581	5270	3503	— 1714	— 8901
80°	— 8920	— 4663	— 924	1585	992	2497	5413	3674	3943	4156	134	— 8579
90°	— 9800	— 5497	— 1804	905	1459	3426	5659	3117	3193	5306	1064	— 8601
100°	— 10362	— 6370	— 2880	— 312	1157	3988	5840	3760	3884	6386	1045	— 7873
110°	— 10634	— 7546	— 4131	— 2123	480	4402	5931	4735	4857	7091	2815	— 6760
120°	— 10217	— 9051	— 6093	— 4442	— 584	4400	6249	5293	5615	8031	3881	— 5501
130°	— 9192	— 9873	— 8281	— 6647	— 1778	3976	6314	5642	6380	9713	5310	— 4301
140°	— 8814	— 10019	— 9604	— 8291	— 3145	3221	6025	5841	7321	11107	6917	— 2857
150°	— 7375	— 9720	— 10159	— 9240	— 3255	2047	5320	5907	8460	12349	8224	— 992

Z.

i	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	46564	47470	52326	55143	55994	51210	50096	56265	63181	57355	58943	51254
40°	42791	41376	45359	50137	52300	45169	41086	50595	58197	63022	58287	47611
50°	37500	34740	38037	42549	43701	36470	35100	41417	50698	57624	53404	43124
60°	29764	25355	28025	32512	32819	26481	27156	33508	40909	48390	47750	37833
70°	17432	12249	13670	17254	19874	16643	18614	24823	29793	35548	37572	31382
80°	5355	— 696	— 154	1564	5777	5670	9056	13630	16037	21228	25448	21651
90°	— 4056	— 13283	— 13613	— 12602	— 9029	— 7731	— 2093	1025	2479	7297	13315	11446
100°	— 11257	— 22887	— 25732	— 25596	— 23552	— 21898	— 14812	— 11372	— 8815	— 4337	3187	2301
110°	— 17059	— 28453	— 34809	— 36591	— 38752	— 36330	— 28714	— 33410	— 20472	— 13584	— 5228	— 4813
120°	— 21558	— 32048	— 40126	— 43907	— 49840	— 48918	— 39947	— 35361	— 31285	— 22585	— 13063	— 11064
130°	— 25529	— 34730	— 43498	— 49757	— 56560	— 56510	— 50131	— 44263	— 40447	— 32384	— 19476	— 18010
140°	— 31275	— 35362	— 46356	— 54530	— 65722	— 65348	— 57005	— 51563	— 49300	— 41915	— 28681	— 25783
150°	— 37016	— 42745	— 49083	— 57514	— 64712	— 62761	— 63853	— 63033	— 60636	— 52203	— 38040	— 34363

[1]) Den Zahlen dieser wie aller folgenden Tabellen liegt die Einheit $0{,}1^{5}\,cm^{-\frac{1}{2}}\,g^{\frac{1}{2}}\,s^{-1}$ zu Grunde.

VI a. Beobachtete Werte der Koeffizienten k und K in der Entwickelung von

X.

u	k_0	k_1	K_1	k_2	K_2	k_3	K_3	k_4	K_4
30°	12930	—1590	5180	2820	1110	—50	—70	—130	—10
35°	15690	—2290	4970	2470	1240	—100	140	—90	—50
40°	18530	—2050	4820	1590	1250	120	230	—50	—60
45°	21300	—2640	4720	590	1280	360	440	20	—70
50°	23930	—2360	4570	—440	1330	510	690	110	—100
55°	26520	—2010	4410	—1420	1320	560	790	270	10
60°	28830	—1710	3970	—2210	1350	360	830	340	70
65°	30770	—1550	3500	—2660	1320	190	770	350	40
70°	32290	—1620	3030	—2820	1190	140	710	330	60
75°	33400	—1960	2470	—2650	960	70	820	350	70
80°	34070	—2470	1930	—2400	640	440	620	400	90
85°	34160	—3240	1630	—1980	150	460	480	370	110
90°	33700	—4220	1380	—1720	—280	310	380	200	160
95°	32730	—5110	1100	—1500	—610	140	170	230	160
100°	31540	—5830	660	—1380	—890	110	40	120	160
105°	30110	—6380	70	—1140	—1320	50	30	70	40
110°	28600	—6430	—570	—780	—1340	—40	—10	—190	—20
115°	26850	—5900	—1530	—530	—1350	—140	—130	—260	—10
120°	25160	—5080	—2600	—420	—1290	—250	—230	—270	60
125°	23580	—3970	—3760	—470	—1260	—360	—320	—210	170
130°	22090	—2620	—4800	—440	—1220	—420	—460	—170	180
135°	20600	—1070	—5730	—340	—1260	—500	—580	—140	200
140°	19140	490	—6580	—130	—1270	—630	—610	—170	200
145°	17480	2210	—7350	150	—1130	—740	—600	—230	160
150°	15720	3760	—7710	420	—800	—720	—600	—230	140

VI b. Beobachtete Werte der Koeffizienten l und L in der Entwickelung von

Y.

u	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
30°	40	—4350	1190	—1070	4910	—90	—510	300	—210
35°	60	—4620	770	—1090	5190	—180	—630	440	—210
40°	110	—4780	350	—1230	5220	—220	—650	590	—300
45°	150	—4850	20	—1420	4960	—300	—550	700	—380
50°	150	—4920	—230	—1580	4500	—400	—360	780	—380
55°	120	—5050	—410	—1710	3980	—480	—150	850	—300
60°	100	—5180	—520	—1850	3390	—560	90	870	—150
65°	50	—5300	—570	—1980	2740	—710	310	780	20
70°	0	—5400	—630	—2160	2110	—930	500	700	170
75°	—50	—5530	—670	—2320	1500	—1190	680	620	310
80°	—110	—5670	—750	—2500	1000	—1420	820	570	450
85°	—150	—5850	—870	—2600	580	—1600	1030	520	530
90°	—130	—6010	—1130	—2650	240	—1700	1170	510	570

u	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
95°	—110	—6170	—1500	—2650	—10	—1750	1270	490	570
100°	—80	—6370	—2040	—2610	—130	—1720	1320	390	530
105°	—60	—6520	—2050	—2460	—260	—1670	1340	270	440
110°	—80	—6670	—3300	—2310	—450	—1030	1320	170	330
115°	—150	—6770	—4030	—2110	—690	—1580	1310	130	250
120°	—210	—6820	—4810	—1910	—940	—1530	1330	160	200
125°	—250	—6780	—5650	—1710	—1170	—1460	1390	190	190
130°	—280	—6690	—6490	—1540	—1340	—1390	1510	210	220
135°	—290	—6520	—7270	—1400	—1450	—1350	1600	190	210
140°	—200	—6230	—7940	—1360	—1530	—1330	1630	190	180
145°	—60	—5830	—8520	—1390	—1620	—1190	1600	170	200
150°	140	—5390	—8980	—1360	—1700	—950	1470	150	260

VI c. Beobachtete Werte der Koeffizienten m und M in der Entwickelung von

Z.

u	m_0	m_1	M_1	m_2	M_2	m_3	M_3	m_4	M_4
30°	53950	—2530	—3250	—4910	220	900	—720	—990	—110
35°	52470	—2380	—4540	—5770	—500	850	—280	—510	320
40°	50010	—1750	—5800	—6820	—1120	970	170	—60	750
45°	46810	—830	—6590	—7300	—1290	1130	500	180	590
50°	42940	170	—6950	—7210	—1670	1280	—20	400	700
55°	38850	740	—7510	—6840	—2110	520	—60	540	920
60°	34270	1160	—8430	—6320	—2580	0	—640	250	810
65°	28750	800	—9450	—5350	—3190	—280	—1270	—250	540
70°	22920	240	—10230	—4200	—3600	—460	—1530	—720	400
75°	16760	—160	—10810	—3200	—4060	—860	—1690	—850	270
80°	10430	—10	—11040	—2040	—4360	—1220	—1850	—1040	170
85°	4050	340	—11280	—1110	—4670	—1440	—1980	—990	10
90°	—2250	860	—11660	—340	—4530	—1460	—2260	—710	—70
95°	—8220	1820	—12280	170	—4280	—1250	—2480	—490	—140
100°	—13770	3180	—12810	700	—4200	—1090	—2440	—350	—150
105°	—19050	5060	—13450	1220	—3780	—1000	—2360	—60	20
110°	—24030	7380	—13850	1400	—3200	—1230	—2220	140	20
115°	—28430	9560	—13590	1390	—2840	—1630	—2170	450	60
120°	—32450	11520	—13180	1310	—2450	—1690	—2240	380	290
125°	—35980	12820	—12130	1360	—2100	—1560	—2240	150	510
130°	—39380	13870	—11150	1590	—1680	—1650	—2030	—180	410
135°	—43030	14800	—10680	2100	—350	—2360	—2200	240	90
140°	—46370	15220	—9140	2260	60	—2360	—2370	70	280
145°	—49030	14830	—6330	2020	—950	—2000	—1730	—320	200
150°	—52210	14780	—3710	2120	—2160	—1920	—700	—390	—170

VII. Koeffizienten der zur Darstellung der Kraftkomponenten dienenden Reihen.

	$\alpha X \sin v$ I	II	III	IV	$\beta Y \sin v$ I	II	III	IV	γZ
ρ_0^0	21093.2	21255.0	21278.4	21278.2	0	− .7	0	− 50.0	0
ρ_0^1	363.4	323.0	361.1	361.1	0	139.4	0	66.7	36388.2
ρ_0^2	−10152.6	−10281.4	−10234.7	−10234.7	0	93.4	0	34.1	701.4
ρ_0^3	− 881.7	− 980.1	− 932.4	− 932.0	0	39.7	0	− 37.4	− 1412.1
ρ_0^4	682.7	649.4	695.1	695.3	0	49.2	0	− 9.1	− 1272.6
ρ_0^5	493.4	494.7	534.4	534.4	0	128.1	0	− 4.8	291.4
ρ_0^6	− 122.1	− 167.6	− 132.9	− 132.9	0	44.1	0	0.2	− 40.0
ρ_0^7	17.2	− 7.0	17.7	17.7					
ρ_1^1	−1724.7	−1932.3	−1831.0	−1920.9	−3456.3	−3417.2	3414.6	−3421.2	2847.6
q_1^1	433.0	436.1	431.1	438.4	−1430.4	−1279.2	−1294.2	−1276.2	− 6880.6
ρ_1^2	272.1	270.4	268.6	258.4	323.2	327.2	321.2	329.2	− 3840.6
q_1^2	1759.1	1752.6	1737.0	1735.2	1285.3	1272.1	1365.4	1260.2	965.0
ρ_1^3	784.1	752.1	814.7	788.3	111.6	145.2	110.2	131.6	1899.7
q_1^3	− 545.0	− 449.4	− 457.4	− 442.0	− 476.2	− 505.2	− 443.2	− 495.4	444.3
ρ_1^4	− 997.7	− 899.0	− 956.1	− 924.4	− 97.0	− 125.6	− 61.7	− 120.8	− 908.1
q_1^4	91.0	162.2	112.7	128.7	182.4	189.1	201.2	166.1	− 483.4
ρ_1^5	293.4	172.0	274.7	226.9	86.1	91.2	92.8	61.2	− 606.7
q_1^5	90.2	− 1.2	16.2	10.1	101.4	147.0	104.2	169.7	514.6
ρ_1^6	251.1	313.4	257.4	273.0	− 28.0	− 14.2	− 30.4	− 6.8	− 132.0
q_1^6	− 212.9	− 167.0	− 229.4	− 221.3	18.6	− 1.7	35.2	− 38.7	− 201.7
ρ_1^7	56.2	− 13.0	104.0	54.6					
q_1^7	86.1	81.4	90.2	95.6					

	$\alpha X \sin v$ I	II	III	IV	$\beta Y \sin v$ I	II	III	IV	γZ
ρ_2^2	−831.4	−784.2	−792.1	782.7	−1333.1	−1246.3	−1257.4	−1246.4	− 959.2
q_2^2	− 20.4	− 54.2	− 11.2	− 54.1	642.4	481.4	494.6	482.4	−1989.2
ρ_2^3	−106.1	−174.0	−203.1	−179.2	− 27.1	22.4	− 15.6	22.3	−2190.0
q_2^3	662.4	607.6	582.4	605.7	1100.4	1054.0	1047.4	1051.4	− 54.2
ρ_2^4	531.0	387.4	393.1	395.2	145.4	96.3	8.4	94.6	− 796.4
q_2^4	61.3	49.1	33.4	49.2	320.4	366.0	9357.4	372.4	362.2
ρ_2^5	374.7	385.2	361.2	372.4	31.4	− 2.1	17.1	− 1.4	− 409.6
q_2^5	−170.4	−109.3	−136.7	−113.4	137.1	233.2	212.0	215.6	94.4
ρ_2^6	[illegible]	221.4	250.7	252.1	− 22.0	− 57.4	− 27.2	− 63.7	168.4
q_2^6	− 37.4	20.0	− 20.3	− 18.6	− 48.1	− 21.3	− 19.4	− 2.0	− 76.7
ρ_2^7	− 69.4	13.0	− 28.0	− 20.7					
q_2^7	31.7	62.4	40.1	52.0					
ρ_3^3	173.0	72.4	158.8	70.7	− 821.7	− 749.2	− 740.2	− 749.3	− 582.4
q_3^3	144.4	166.3	119.8	165.4	439.2	516.0	506.2	517.4	−1089.4
ρ_3^4	134.4	129.6	150.8	125.1	260.0	204.4	214.2	202.2	516.2
q_3^4	240.7	226.3	246.1	226.4	− 311.4	− 317.2	− 286.6	− 311.6	431.2
ρ_3^5	17.4	− 70.4	− 50.2	− 84.6	− 41.1	11.1	− 0.0	12.3	− 29.4
q_3^5	− 59.0	− 51.6	− 65.2	− 57.0	14.0	− 13.1	22.4	5.2	− 82.0
ρ_3^6	10.8	48.4	16.3	14.1	78.3	32.4	49.0	16.7	423.2
q_3^6	29.1	− 2.0	0.7	− 0.0	− 182.1	− 123.2	− 101.4	− 80.4	182.0
ρ_3^7	−165.0	5.2	92.0	− 98.1					
q_3^7	− 70.0	− 0.6	− 44.6	− 28.3					
ρ_4^4	− 26.8	83.0	44.4	84.2	91.7	273.0	239.2	275.0	− 189.2
q_4^4	35.1	45.8	48.3	42.4	152.3	160.3	181.0	160.0	114.0
ρ_4^5	99.0	104.2	42.4	107.1	77.4	129.2	106.2	108.0	− 88.3
q_4^5	47.0	− 17.4	−100.2	− 13.0	59.4	− 81.0	− 98.7	− 83.3	115.2
ρ_4^6	27.2	− 51.2	− 46.1	− 44.2	115.3	− 107.4	− 42.0	− 91.2	141.2
q_4^6	− 36.2	− 1.0	− 49.3	− 49.7	− 81.3	67.0	19.1	54.4	201.2
ρ_4^7	− 50.0	− 11.6	11.2	14.7					
q_4^7	− 71.2	− 1.2	26.4	30.2					

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XIII a. Differenzen der nach den 4 Hypothesen I, II, III, IV der Tabelle VII berechneten Werte von k_2 und K_1 mit den beobachteten.[1])

u	k_2 I	k_2 II	k_2 III	k_2 IV	K_1 I	K_1 II	K_1 III	K_1 IV
30°	59	418	9	291	650	165	540	403
35°	− 94	112	− 15	131	388	− 29	180	91
40°	−266	− 87	− 88	− 5	228	−121	− 43	− 96
45°	−341	−139	−144	− 62	185	− 97	−107	−137
50°	−361	− 92	−182	− 60	178	− 37	− 91	−108
55°	−406	− 85	−254	− 78	253	108	47	37
60°	−459	− 40	−320	−100	129	59	17	11
65°	−487	− 39	−335	− 95	34	42	30	30
70°	−491	− 27	−298	− 67	− 37	53	69	76
75°	−475	− 27	−224	− 23	−206	− 38	− 6	10
80°	−335	68	− 46	114	−381	−147	−117	− 93
85°	−291	109	53	179	−345	− 66	− 55	− 25
90°	−342	48	0	109	−262	30	13	43
95°	−337	52	− 35	74	−167	99	54	77
100°	−325	55	− 98	25	−132	65	5	14
105°	−417	− 67	−281	−144	− 88	− 2	− 54	− 65
110°	−367	− 81	−313	−175	99	44	26	− 7
115°	−139	37	−137	− 21	161	− 48	− 9	− 63
120°	− 30	37	− 34	38	264	− 86	20	− 50
125°	− 11	− 75	26	46	346	−107	56	− 21
130°	− 80	−144	38	21	498	9	194	121
135°	−198	−164	15	9	578	140	292	232
140°	−471	−161	−174	− 91	440	151	196	159
145°	−659	134	−317	− 44	2	− 37	−181	−189
150°	−996	480	−665	− 99	−428	−133	−542	−517

XIII b. Differenzen der nach den 4 Hypothesen I, II, III, IV der Tabelle VII berechneten Werte von l_2 und L_1 mit den beobachteten.

u	l_2 I	l_2 II	l_2 III	l_2 IV	L^1 I	L^1 II	L^1 III	L^1 IV
30°	224	93	− 20	213	234	− 16	−543	66
35°	59	− 43	−129	33	234	16	−439	29
40°	17	− 68	−118	− 33	185	1	336	− 30
45°	73	− 6	− 15	− 7	161	6	−344	− 42
50°	133	50	84	22	146	11	−299	− 31
55°	131	40	111	− 6	132	5	−267	− 17
60°	125	23	121	− 29	134	0	−229	4
65°	122	15	124	− 34	163	6	−171	32
70°	132	28	129	− 10	175	− 21	−130	18
75°	111	20	95	− 2	228	− 17	− 49	21
80°	84	14	47	10	287	− 11	− 41	13
85°	25	− 15	− 36	− 5	405	29	159	30
90°	0	− 8	− 86	11	406	25	321	1
95°	− 11	12	−118	31	390	22	263	− 20
100°	− 48	3	−172	7	336	− 35	301	− 83
105°	− 31	22	−166	16	311	− 51	189	− 88
110°	− 24	24	−160	− 4	301	− 11	185	− 22
115°	9	33	−122	− 16	272	22	155	47
120°	45	31	− 70	− 33	235	48	111	109
125°	107	43	14	− 25	163	27	33	112
130°	138	17	73	− 37	95	− 12	− 35	70
135°	158	− 20	124	− 40	75	− 38	− 46	5
140°	210	− 23	209	14	133	− 23	34	− 65
145°	290	11	321	125	236	− 4	174	−175
150°	348	32	409	242	397	− 42	387	−299

1) Um für alle folgenden Tabellen eine zweckmässige Anordnung zu gewinnen, habe ich Tabelle XIII an dieser Stelle, an der sie übrigens auch ihrem Inhalte nach allenfalls stehen kann, eingeschoben. Eine Aenderung der Numerierung war leider nicht mehr möglich, da der Druck des Textes abgeschlossen war.

VIII. Endgültig gewählte Koeffizienten der Reihen zur Darstellung von $\alpha X \sin v$, $\beta Y \sin v$, γZ.

$\alpha X \sin v$

m; n:	0	1	2	3	4	5	6	7
0	21280.43	360.49	−10235	− 933	695	535	−133	18
1		−1920.01 437.42	259 1736	789 − 432	−924 128	227 10	274 −221	55 96
2			− 781.34 − 55.44	− 380 606	396 50	373 −113	252 − 19	− 21 52
3				71.11 166.31	125 227	− 85 − 57	14 − 1	− 68 − 28
4					84.31 41.40	107 − 14	− 45 − 50	15 31

$\beta Y \sin v$

m; n:	0	1	2	3	4	5	6
0	− 49.00	66.30	34	− 37	− 9	− 5	0
1		−3420.31 −1276.46	329 1261	132 − 495	− 121 166	61 170	− 7 − 38
2			−1247.10 482.35	23 1051	94 372	− 1 216	− 64 − 2
3				− 748.31 516.30	202 − 312	12 5	16 − 80
4					274.00 160.34	109 − 84	− 92 55

γZ

m; n:	0	1	2	3	4	5	6
0	0	36388	701	−1412	−1273	291	− 40
1		2847 − 6881	−3841 966	1900 445	− 909 − 483	−607 515	−132 −202
2			− 959 −1990	−2191 − 54	− 797 362	−410 95	168 − 77
3				− 583 −1089	517 431	− 30 − 82	423 182
4					− 190 114	− 89 116	141 201

IX. Koeffizienten der Reihen zur Darstellung von U, W und $V:b$.

($b = 6{,}356 \cdot 10^8\ cm$)

U_0

m; n:	0	1	2	3	4	5	6
0	0	−18429,4	−232,1	354,0	276,4	−53,0	6,0
1		−1164,1	1202,1	−407,2	147,4	110,1	18,4
		3484,2	−304,4	−92,9	61,8	−89,1	32,3
2			308,4	527,4	189,4	106,4	−7,1
			718,1	21,3	−28,0	−8,0	18,0
3				140,3	−100,1	6,4	−25,0
				225,9	−54,0	−0,4	−10,3
4					101,9	−24,1	6,3
					15,7	−26,7	12,4

$U = U_0 + \Pi_1 \cdot (-532{,}0 \cos\lambda + 50{,}1 \sin\lambda) + \Pi_2 \cdot (30{,}9 \cos 2\lambda - 45{,}4 \sin 2\lambda) +$
$\Pi_3 \cdot (-200{,}3 \cos 3\lambda + 156{,}2 \sin 3\lambda) + \Pi_4 \cdot (90{,}2 \cos 4\lambda - 15{,}9 \sin 4\lambda)$.

W_0

m; n:	0	1	2	3	4	5	6
0	0	−18429,4	−232,1	354,0	276,4	−53,0	6,0
1		−1276,1	1261,0	−495,0	166,0	170,4	−38,0
		3421,0	−329,0	−132,0	121,0	−61,0	7,0
2			241,2	525,4	186,0	108,0	−1,0
			623,0	−11,3	−47,0	0,5	32,0
3				172,1	−104,0	1,1	−26,7
				249,4	−67,4	−4,0	−5,3
4					40,1	−21,0	13,4
					−68,3	−27,4	23,0

$W = W_0 - i\,(-49{,}00 + 66{,}11\, R_0^1 + 34\, R_0^2 - 37\, R_0^3 - 9\, R_0^4 - 5\, R_0^5)$

$V:b$

m; n:	0	1	2	3	4	5	6
0	0	−18429,4	−232,1	354,9	276,4	−53,0	6,0
1		−1220,4	1231,4	−451,4	156,0	140,4	−9,4
		3452,0	−316,8	−112,4	91,4	−75,3	19,4
2			274,4	526,4	187,7	107,1	−4,2
			671,1	4,0	−38,0	−3,4	25,0
3				156,4	−102,3	4,2	−25,4
				237,4	−61,2	−2,3	−7,4
4					71,0	−22,0	9,0
					−26,4	−27,0	17,0

X. Koeffizienten der Reihen zur Darstellung von $V_e:b$, $V_a:b$ und $\alpha\beta b i$.

($b = 6{,}356\ 10^8\ cm$)

$V_e:b$

m; n:	0	1	2	3	4	5	6
0	0	−18321,2	−233,4	354,2	264,0	−50,4	5,0
1		−1360,4	1264,0	−465,0	171,1	119,3	5,2
		3455,4	−320,1	−112,0	94,4	−81,3	24,4
2			302,4	540,0	172,4	86,3	−14,0
			668,4	9,0	−57,3	−10,4	17,4
3				150,1	−103,2	4,4	−44,4
				258,3	−75,3	6,4	−17,3
4					52,4	−2,3	−6,3
					−24,4	−22,0	−7,3

$V_a:b$

m; n:	0	1	2	3	4	5	6
0	0	−108,1	1,1	−0,3	11,7	−2,4	0,1
1		140,0	−32,4	14,4	−14,2	21,1	−15,3
		−2,4	3,0	−0,4	−3,2	6,4	−5,0
2			−27,0	−13,4	15,3	21,2	10,4
			2,4	−5,0	19,2	6,4	7,3
3				5,4	1,0	−0,3	18,4
				−20,7	14,2	−8,7	9,0
4					18,3	−20,4	16,3
					−1,0	−4,3	25,1

$\alpha\beta b i$

m; n:	0	1	2	3	4	5
0	0	6,4	−20,4	−5,7	−4,0	0,0
1		−3,4	−2,0	−15,4	−22,0	24,4
		−33,2	3,4	−39,3	47,4	−54,4
2			12,2	−16,2	−14,7	−28,3
			2,0	14,2	2,4	12,7
3				−5,7	3,4	5,4
				−9,4	−4,7	−1,4
4					−0,1	−13,4
					4,2	9,4

XI a. Berechnete Werte der Koeffizienten k und K in der Entwickelung von

X.

u	k_0	k_1	K_1	k_2	K_2	k_3	K_3	k_4	K_4
0°	0,0	2579,1	4257,6	0,0	0,0	0,0	0,0	0,0	0,0
5°	1907,1	2371,6	4273,3	1170,2	260,1	−21,5	−2,3	0,3	0,3
10°	3870,1	1783,3	4322,1	2202,3	501,1	−80,2	−6,3	3,8	1,0
15°	5939,3	915,3	4403,8	2975,0	705,4	−158,3	−4,9	11,3	2,8
20°	8149,4	−89,3	4515,1	3404,3	866,9	−231,3	10,6	26,3	4,7
25°	10520,3	−1070,0	4646,4	3446,0	988,1	−275,5	49,0	47,4	5,0
30°	13047,1	−1881,4	4777,4	3107,3	1079,2	−272,2	115,2	74,9	1,9
35°	15712,8	−2420,6	4878,9	2442,3	1153,8	−214,8	210,9	107,6	−6,2
40°	18463,1	−2645,2	4916,4	1541,8	1223,8	−108,7	328,3	143,2	−19,2
45°	21235,8	−2577,7	4856,9	519,8	1291,2	29,2	456,3	181,0	−35,9
50°	23947,8	−2299,6	4677,4	−500,0	1352,8	174,0	579,7	220,6	−52,1
55°	26508,8	−1931,9	4373,6	−1408,4	1392,2	303,2	683,8	256,7	−63,1
60°	28822,6	−1609,8	3958,7	−2116,6	1387,2	393,0	754,2	289,9	−63,4
65°	30709,3	−1454,8	3470,1	−2573,3	1314,8	435,6	783,8	317,4	−48,0
70°	32359,8	−1552,7	2954,3	−2766,0	1157,6	428,7	769,4	336,8	−17,8
75°	33444,6	−1937,0	2450,3	−2720,4	909,0	383,8	714,9	354,1	27,9
80°	34019,0	−2584,2	2023,2	−2490,7	578,7	316,2	628,2	336,7	82,7
85°	34081,1	−3419,3	1655,0	−2148,8	190,6	244,6	519,1	311,8	138,2
90°	33655,6	−4329,3	1337,8	−1708,7	−219,8	181,2	398,8	266,8	184,6
95°	32795,6	−5184,0	1023,2	−1415,8	−611,0	133,6	273,7	202,9	211,8
100°	31594,0	−5855,2	645,7	−1132,0	−945,3	95,8	150,8	122,4	213,8
105°	30135,1	−6236,3	134,9	−935,3	−1194,6	58,0	29,6	30,8	187,2
110°	28524,0	−6254,6	−562,8	−816,2	−1345,3	5,7	−87,3	−64,4	135,0
115°	26853,6	−5878,8	−1460,8	−745,0	−1401,0	−73,2	−200,8	−154,0	64,8
120°	25199,7	−5117,9	−2550,3	−680,7	−1383,1	−184,1	−308,1	−229,2	−13,6
125°	23600,7	−4015,2	−3739,4	−553,4	−1316,8	−322,8	−405,9	−282,2	−87,8
130°	22072,0	−2641,3	−4920,8	−424,8	−1232,2	−471,3	−486,9	−307,8	−144,9
135°	20589,2	−1079,1	−5902,4	−196,1	−1154,0	−609,3	−542,7	−305,8	−179,8
140°	19095,8	581,4	−6739,2	88,6	−1094,8	−709,6	−565,2	−277,7	−187,9
145°	17516,2	2253,8	−7161,0	393,0	−1053,1	−750,6	−549,2	−231,9	−172,2
150°	15772,7	3858,9	−7192,8	672,4	−1016,3	−721,8	−494,4	−174,2	−139,8
155°	13794,3	5328,0	−6867,6	872,9	−963,1	−623,2	−406,1	−117,3	−98,9
160°	11534,1	6606,4	−6281,0	953,7	−872,3	−474,7	−298,7	−67,5	−59,3
165°	8980,1	7646,8	−5577,8	891,8	−728,2	−304,9	−186,9	−31,2	−28,2
170°	6158,8	8414,3	−4918,7	689,6	−526,0	−148,8	−89,0	−9,9	−9,3
175°	3134,1	8854,6	−4453,9	375,8	−277,2	−39,8	−23,4	−1,8	−1,2
180°	0,0	9043,1	−4256,6	0,0	0,0	0,0	0,0	0,0	0,0

XI b. Berechnete Werte der Koeffizienten l und L in der Entwickelung von

Y.

u	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
0°	0,0	−4257,8	2579,6	0,0	0,0	0,0	0,2	0,0	0,0
5°	33,0	−4267,4	2530,4	−261,3	1182,9	2,3	−21,9	−0,1	0,1
10°	64,0	−4296,3	2386,0	−500,3	2300,7	7,0	−83,8	−1,9	3,6
15°	91,3	−4342,8	2155,0	−720,2	3293,4	13,4	−173,8	−2,2	11,4
20°	113,3	−4404,6	1855,0	−896,3	4111,7	13,8	−275,3	−2,4	24,8
25°	129,0	−4478,9	1505,2	−1033,1	4719,1	3,1	−369,0	1,2	43,2
30°	137,6	−4562,8	1123,7	−1136,0	5095,7	−24,2	−438,1	15,1	65,2
35°	138,8	−4651,8	741,4	−1219,3	5238,9	−74,7	−464,7	42,6	87,0
40°	132,8	−4746,7	380,4	−1297,2	5161,7	−149,7	−446,4	88,7	108,8
45°	120,3	−4843,2	61,0	−1384,8	4891,7	−251,0	−363,8	156,6	124,8
50°	102,2	−4942,4	−198,0	−1494,0	4466,2	−377,0	−236,7	245,8	134,8
55°	79,0	−5044,3	−393,2	−1634,4	3930,8	−523,0	−72,2	352,8	138,0
60°	54,2	−5151,2	−523,8	−1802,7	3330,8	−685,8	115,0	469,8	139,3
65°	27,7	−5265,8	−601,4	−1991,0	2710,6	−856,1	312,4	587,0	138,7
70°	0,2	−5390,4	−647,7	−2187,8	2108,2	−1027,0	503,4	691,9	141,7
75°	−26,8	−5528,3	−690,2	−2371,0	1552,0	−1191,2	678,2	771,8	152,8
80°	−52,3	−5679,2	−763,4	−2523,0	1063,7	−1342,8	830,8	815,0	174,5
85°	−75,5	−5845,2	−900,2	−2626,4	648,9	−1473,8	960,2	816,2	209,6
90°	−96,2	−6021,3	−1131,8	−2669,7	307,2	−1582,2	1069,8	770,2	257,6

Y.

u	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
95°	−115,3	−6201,3	−1480,0	−2648,3	29,4	−1063,8	1164,0	680,3	315,4
100°	−131,7	−6376,4	−1957,4	−2565,0	−198,3	−1716,4	1249,4	553,0	378,2
105°	−145,2	−6536,0	−2562,3	−2432,9	−392,0	−1730,2	1330,3	403,2	439,2
110°	−156,4	−6666,2	−3277,0	−2266,3	−569,0	−1731,6	1406,9	243,3	490,3
115°	−165,4	−6754,1	−4076,4	−2084,3	−738,8	−1691,7	1473,0	80,2	525,0
120°	−171,7	−6787,9	−4919,2	−1906,1	−907,0	−1621,4	1525,9	−44,7	539,0
125°	−175,8	−6755,1	−5762,0	−1744,2	−1076,8	−1521,0	1551,0	−147,2	537,7
130°	−176,4	−6653,0	−6560,2	−1608,8	−1238,1	−1394,6	1540,8	−213,3	491,6
135°	−174,2	−6480,4	−7274,6	−1498,8	−1380,8	−1243,2	1482,0	−240,9	433,8
140°	−168,8	−6243,4	−7875,3	−1406,7	−1488,4	−1072,2	1373,0	−235,1	360,2
145°	−159,3	−5954,6	−8344,9	−1322,2	−1545,6	−888,8	1212,9	−203,7	279,6
150°	−146,3	−5631,3	−8681,4	−1229,6	−1537,3	−700,2	1010,6	−157,0	199,2
155°	−129,2	−5297,4	−8895,8	−1113,8	−1452,2	−516,6	781,2	−107,2	128,3
160°	−108,6	−4976,7	−9010,3	−963,3	−1288,3	−347,6	546,6	−62,8	71,2
165°	−84,6	−4694,8	−9054,3	−771,8	−1026,8	−203,2	329,0	−29,0	32,2
170°	−58,0	−4474,6	−9058,0	−541,8	−738,6	−93,0	153,7	−9,3	10,0
175°	−29,8	−4334,6	−9048,3	−270,2	−382,3	−23,6	39,7	−1,3	1,2
180°	0,0	−4286,3	−9043,2	0,0	0,0	0,0	0,0	0,0	0,0

XI c. Berechnete Werte der Koeffizienten m und M in der Entwickelung von

Z.

u	m_0	m_1	M_1	m_2	M_2	m_3	M_3	m_4	M_4
0°	57859,2	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
5°	57788,7	−1386,2	−578,1	−180,3	−6,8	8,6	2,7	0,1	0,3
10°	57563,7	−2572,0	−1159,2	−704,1	−28,3	62,1	19,1	1,2	4,2
15°	57147,6	−3391,0	−1750,3	−1523,3	−66,8	190,3	60,4	5,8	20,0
20°	56483,2	−3748,2	−2362,6	−2559,1	−127,3	410,8	121,0	16,2	58,0
25°	55499,1	−3624,1	−3012,2	−3712,2	−218,0	670,6	190,0	32,6	126,2
30°	54116,0	−3086,0	−3714,0	−4864,3	−353,8	951,4	244,6	52,0	227,8
35°	52255,8	−2266,3	−4480,2	−5895,0	−544,6	1101,3	255,4	70,8	354,2
40°	49848,8	−1332,0	−5307,2	−6692,4	−806,3	1249,0	199,3	77,3	492,9
45°	46843,8	−435,2	−6179,2	−7167,2	−1148,0	1177,9	63,0	62,1	621,8
50°	43210,0	223,2	−7066,8	−7264,8	−1571,2	938,2	−156,2	16,6	717,7
55°	38951,2	618,6	−7930,1	−6973,2	−2065,7	559,4	−443,2	−63,4	760,1
60°	34095,2	717,3	−8731,2	−6326,0	−2605,4	99,1	−772,4	−175,6	737,2
65°	28709,1	577,2	−9439,8	−5397,1	−3163,3	−365,2	−1110,6	−309,2	648,3
70°	22854,9	311,8	−10044,4	−4290,2	−3656,0	−771,7	−1425,8	−449,8	504,7
75°	16743,8	65,1	−10555,0	−3122,6	−4128,7	−1038,2	−1692,6	−575,2	370,6
80°	10422,6	1,1	−11001,8	−2005,3	−4447,0	−1206,2	−1898,6	−664,4	150,7
85°	4067,7	242,6	−11421,0	−1028,8	−4610,8	−1229,2	−2044,6	−701,2	−2,6
90°	−2178,1	881,1	−11851,9	−247,0	−4601,8	−1171,3	−2142,8	−676,0	−107,0
95°	−8157,2	1948,8	−12311,0	324,3	−4426,1	−1093,9	−2211,7	−588,9	−149,2
100°	−13858,1	3417,2	−12784,0	708,4	−4106,0	−1058,7	−2269,2	−450,6	−128,2
105°	−19123,2	5205,2	−13220,7	954,8	−3682,0	−1110,6	−2325,7	−279,8	−55,8
110°	−23953,8	7189,9	−13535,2	1124,4	−3201,2	−1264,4	−2381,2	−100,1	48,7
115°	−28361,8	9225,9	−13622,2	1274,7	−2708,9	−1502,5	−2423,3	64,3	160,2
120°	−32359,8	11159,0	−13375,9	1443,2	−2246,6	−1775,7	−2431,6	194,0	255,4
125°	−36107,0	12848,3	−12716,7	1639,7	−1841,4	−2019,2	−2380,9	276,2	317,2
130°	−39589,8	14167,8	−11613,7	1845,8	−1505,2	−2168,8	−2252,2	308,1	337,7
135°	−42908,3	15021,2	−10099,0	2020,3	−1235,2	−2175,0	−2037,2	295,2	318,9
140°	−46113,0	15341,5	−8276,1	2114,2	−1018,7	−2021,3	−1743,2	249,2	267,9
145°	−49219,4	15093,4	−6290,2	2084,8	−835,8	−1724,8	−1393,7	188,6	201,2
150°	−52204,6	14260,2	−4358,2	1910,2	−670,7	−1334,2	−1024,6	124,2	133,7
155°	−55005,8	12889,3	−2639,7	1599,0	−512,8	−915,0	−670,1	70,8	76,2
160°	−57528,6	10998,7	−1293,6	1191,8	−360,3	−535,9	−385,8	33,2	35,7
165°	−59659,7	8607,0	−402,0	754,2	−219,1	−252,6	−176,9	11,8	12,8
170°	−61284,2	5953,2	38,9	364,2	−103,7	−80,8	−55,7	2,6	2,3
175°	−62303,8	3054,6	119,4	95,8	−26,9	−10,8	−7,2	0,3	0,2
180°	−62651,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0

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XII a. Differenzen zwischen den beobachteten und den berechneten Werten der Koeffizienten k und K.

(Beob.-Ber.)

w	k_0	k_1	K_1	k_2	K_2	k_3	K_3	k_4	K_4
30°	−119	291	403	−288	31	222	−186	−205	−12
35°	−22	131	91	28	86	145	−71	−198	−44
40°	67	−5	−96	49	27	229	−98	−194	−41
45°	65	−02	−137	70	−11	331	−16	−162	−34
50°	−18	−60	−108	61	−23	335	110	−110	−48
55°	12	−78	37	−11	−72	257	107	13	73
60°	7	−100	11	−93	−37	−34	76	50	133
65°	−29	−95	30	−87	5	−246	−14	33	89
70°	−70	−67	76	−54	33	−289	−59	−7	78
75°	−45	−23	10	70	51	−313	105	6	42
80°	50	114	−93	91	61	124	−8	63	7
85°	79	179	−25	109	−41	215	−39	58	−28
90°	44	109	43	49	−60	128	−19	−67	−24
95°	−70	74	77	−85	1	7	−104	27	−52
100°	−54	25	14	−248	55	14	−110	−2	−54
105°	−25	−144	−65	−205	−126	−8	0	39	−147
110°	76	−175	−7	36	5	−46	77	−126	−155
115°	−4	−21	−63	215	52	−67	70	−106	−74
120°	−38	38	−50	261	93	−66	78	−41	74
125°	−21	46	−21	113	57	−38	86	72	257
130°	17	21	121	−15	12	51	27	138	325
135°	11	9	232	−144	−106	109	−37	165	379
140°	45	−91	159	−218	−175	80	−45	108	388
145°	−36	−44	−189	−244	−77	11	−51	1	332
150°	−53	−99	−517	−252	216	1	−106	−56	280

XII b. Differenzen zwischen den beobachteten und den berechneten Werten der Koeffizienten l und L.

(Beob.-Ber.)

w	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
30°	−97	213	66	67	−186	−65	−72	285	−275
35°	−79	33	29	130	−49	−105	−165	398	−298
40°	−23	−33	−30	67	58	−70	−210	501	−408
45°	30	−7	−42	−35	68	−49	−187	544	−504
50°	48	22	−31	−85	33	−23	−123	534	−515
55°	40	−6	−17	−76	49	44	−78	497	−439
60°	45	−29	4	−47	59	126	−26	400	−289
65°	22	−34	32	12	29	146	−2	193	−119
70°	0	−10	18	27	2	97	−3	8	28
75°	−23	−2	21	51	−53	1	2	−152	158
80°	−58	10	13	23	−64	−78	−11	−246	276
85°	−74	−5	30	26	−69	−126	76	−296	320
90°	−33	11	1	20	−67	−118	100	−260	312

w	l_0	l_1	L_1	l_2	L_2	l_3	L_3	l_4	L_4
95°	6	31	−20	−2	−39	−86	106	−190	255
100°	52	7	−83	−44	69	−3	70	−164	152
105°	85	16	−88	−27	133	69	10	−133	1
110°	77	−4	−22	−44	119	101	−86	−73	−151
115°	15	−16	47	−25	49	112	−164	41	−276
120°	−38	−33	109	−4	−32	92	−196	205	−339
125°	−75	−25	112	35	−93	62	−162	337	−338
130°	−104	−37	70	69	−102	5	−31	423	−271
135°	−116	−40	5	98	−70	−107	117	431	−224
140°	−31	14	−65	47	−42	−258	257	425	−180
145°	99	125	−175	−68	−74	−302	387	374	−79
150°	286	242	−299	−130	−163	−250	459	308	81

XII c. Differenzen zwischen den beobachteten und den berechneten Werten der Koeffizienten m und M.

(Beob.-Ber.)

w	m_0	m_1	M_1	m_2	M_2	m_3	M_3	m_4	M_4
30°	−166	557	465	−46	573	−51	−904	−1043	−337
35°	214	−114	−60	125	45	−311	−535	−581	−34
40°	161	−418	−493	−128	−314	−280	−30	−137	257
45°	−34	−375	−411	−133	−142	−45	437	118	−32
50°	−271	−53	117	55	−99	342	136	383	−18
55°	−101	121	420	133	−44	−39	383	603	160
60°	174	442	301	6	28	−99	132	426	73
65°	41	223	−10	47	−27	88	−159	60	−108
70°	35	−71	−186	90	86	312	−104	−270	−103
75°	16	−228	−255	−78	69	198	2	−275	−59
80°	7	−11	−39	−35	88	−14	48	−375	19
85°	−18	97	141	−81	−60	−211	64	−289	13
90°	−72	−21	192	−93	72	−289	−117	−34	37
95°	−33	−129	31	−154	146	−156	−268	99	9
100°	88	−237	−26	−8	−93	−31	−171	101	−22
105°	73	−145	−229	265	−97	110	−34	220	76
110°	−70	190	−315	276	1	34	161	240	−29
115°	−09	334	32	115	−131	−128	254	386	−100
120°	−60	360	196	−133	−203	86	191	186	35
125°	127	−29	527	−280	−259	460	141	−126	193
130°	210	−298	464	−250	−175	519	222	−458	72
135°	−122	−221	−580	80	885	−185	−163	−55	−228
140°	−257	−122	−864	146	1078	−339	−627	−180	12
145°	189	−263	−31	−65	−114	−275	−336	−508	−1
150°	−5	511	648	210	−1489	−386	324	−514	−304

XIV a. Abweichungen der berechneten Werte der Komponenten X von den beobachteten.[1])

δX

u \ λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	51	−3	71	−31	20	−81	54	−72	95	−346	255	−155
40°	268	−121	−34	121	−103	123	−145	119	15	−73	203	−293
50°	512	59	162	9	−315	43	−642	37	−322	−206	348	−56
60°	333	−311	193	−158	−26	178	−235	143	54	−171	33	−387
70°	142	−82	−108	50	−31	92	−113	−12	123	−156	147	−302
80°	−183	153	−255	167	−53	−49	74	−36	−29	−3	−467	133
90°	−127	−1	96	−30	117	−286	324	−130	2	57	−744	162
100°	−148	69	50	−102	156	−127	192	−39	−57	282	−448	294
110°	−260	145	−22	−249	164	−43	120	1	−52	−19	−74	335
120°	−86	35	31	−88	62	−154	114	−33	12	−127	−139	80
130°	−11	−44	116	−284	222	−241	41	−14	−56	−182	64	−8
140°	30	−73	25	−116	196	−152	91	−63	100	−219	−66	32
150°	16	0	97	−225	−19	−38	80	−114	114	−235	106	12

$\delta' X$

u \ λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	−99	123	437	553	516	−413	−1125	−412	−108	−625	−335	60
40°	146	26	−116	−174	229	342	−302	327	513	−178	−68	59
50°	208	10	−430	−407	256	237	−342	2	487	29	−287	21
60°	−170	14	−125	85	201	20	98	24	−112	215	150	−316
70°	−487	−76	243	112	−190	−153	225	82	−400	−158	189	−227
80°	442	167	−128	−63	−60	−148	−34	79	167	107	−87	158
90°	263	126	−15	−10	195	83	−211	−68	58	−134	−27	267
100°	−265	−258	176	316	−10	−53	−343	−95	155	68	−37	−303
110°	−235	105	212	−170	18	512	207	−51	307	−2	−53	62
120°	154	344	−90	−468	−293	−12	210	172	−173	−212	−36	−52
130°	212	338	−251	264	372	−246	68	127	−380	76	81	−457
140°	−76	22	−375	575	851	−189	−54	111	−400	167	324	−416
150°	−459	−172	−453	−268	−241	−859	−263	729	544	554	554	−302

ΔX

u \ λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	−48	120	508	522	536	−494	−1071	−484	−13	−971	−80	−95
40°	414	−95	−150	−53	126	465	−447	446	528	−251	135	−234
50°	720	69	−268	−398	−59	280	−984	39	165	−177	−39	−35
60°	163	−297	68	−73	175	198	−137	167	−58	44	183	−703
70°	−345	−158	135	162	−221	−61	112	70	−277	−314	336	−529
80°	259	320	−383	104	−113	−197	40	43	138	104	−554	291
90°	136	125	81	−40	312	−203	113	−198	60	−77	−771	429
100°	−413	−189	226	214	146	−180	−151	−134	98	350	−485	−9
110°	−495	250	190	−419	182	469	327	−50	255	−21	−127	397
120°	68	379	−59	−556	−231	−166	324	139	−161	−339	−175	28
130°	201	294	−135	−20	594	−487	109	113	−436	−106	155	−465
140°	−46	−51	−350	459	1047	−341	37	48	−300	−52	258	−384
150°	−443	−172	−356	−493	−260	−897	−183	615	658	319	660	−290

[1]) Diese Abweichungen sind ebenso wie diejenigen in den vorhergehenden Tabellen im Sinne von (Beobachtung—Rechnung) gebildet.

Tabelle XIII steht zwischen VII und VIII, S. 66.

XIV b. Abweichungen der berechneten Werte der Komponente Y von den beobachteten.

δY

u \ λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	41	17	−11	−73	238	−193	155	−108	226	−336	−93	29
40°	−113	146	−117	10	277	−170	269	−169	99	−182	44	60
50°	−163	267	−262	300	42	−81	371	−299	215	−256	20	18
60°	−97	196	−352	394	−121	−447	319	−387	447	−513	284	−143
70°	110	144	−305	207	−169	−79	346	−455	534	−487	401	−635
80°	210	192	−191	195	−152	42	363	−402	547	−374	521	−623
90°	180	112	−175	175	−93	27	219	−474	255	−24	163	−380
100°	28	80	−107	128	−143	98	50	−223	194	100	4	−130
110°	−114	−16	117	97	−33	172	−149	52	−20	71	−3	−97
120°	93	−184	275	−102	49	173	−141	197	−108	31	−57	−65
130°	198	−219	200	−117	152	102	−156	238	−230	243	−91	−57
140°	116	−139	153	−71	80	−29	−105	250	−244	187	−40	4
150°	35	−170	315	−440	335	−233	50	81	−210	249	−230	24

$\delta' Y$

u \ λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	403	−460	33	259	−464	−30	107	−751	−425	−17	−236	417
40°	442	−797	124	591	−790	−133	648	−290	69	231	−631	260
50°	496	−798	305	759	−712	−2	498	−560	291	575	−590	314
60°	495	−427	33	522	−289	22	301	−329	307	462	−577	20
70°	122	33	−126	2	123	−2	−52	38	46	−40	−115	−31
80°	−310	264	−147	−303	176	−120	−213	250	−335	−351	420	−94
90°	−380	429	−117	−412	293	−14	−166	209	−366	−214	538	−196
100°	−152	338	19	−211	149	−57	−160	269	150	85	306	−102
110°	57	−45	236	152	−23	34	−137	156	500	−16	−191	221
120°	222	−634	113	476	−202	66	104	−294	142	−134	−607	292
130°	356	−632	−167	351	−412	78	420	−576	−241	149	−581	6
140°	197	−176	61	25	−708	185	685	−649	−356	669	−65	−240
150°	456	498	115	−34	−239	255	472	−541	−109	1482	1021	56

ΔY

u \ λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	444	−443	22	186	−226	−223	262	−859	−199	−363	−329	446
40°	329	−651	7	601	−513	−303	917	−459	108	49	−587	320
50°	333	−531	43	1059	−670	−83	869	−859	506	279	−570	332
60°	398	−231	−319	916	−410	−425	620	−716	754	−51	−293	−123
70°	232	177	−431	209	−46	−81	294	−417	590	−527	348	−666
80°	−139	456	−338	−108	124	−78	150	−146	212	−725	941	−717
90°	−200	541	−292	−237	200	13	53	−265	−111	−238	703	−576
100°	−124	418	−88	−93	6	41	−110	46	344	191	348	−232
110°	−57	−61	373	209	−56	206	−286	208	480	55	−194	124
120°	315	−818	388	314	−153	239	−37	−97	34	−103	−664	227
130°	554	−851	93	234	−260	180	264	−338	−471	392	−672	−51
140°	313	−315	214	−46	−628	156	520	−399	−600	856	−105	−236
150°	491	328	430	−474	96	22	522	−460	−219	1731	771	80

Schneller, als ich hoffen konnte, hat sich mir kurz vor dem Abschlusse des Druckes der vorliegenden Arbeit die Gelegenheit zur ausführlichen Veröffentlichung der Rechnung und der Resultate geboten. Herr Geh. R. Neumayer hat mich zu ernstem Danke verpflichtet, indem er den Abdruck

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XIV c. Abweichungen der berechneten Werte der Komponente Z von den beobachteten.

ΔZ

u λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	144	−79	120	−197	29	−531	416	−356	1016	−3045	1679	−795
40°	441	−688	396	−663	408	−567	176	40	−501	282	40	−554
50°	−80	−207	958	−1071	127	137	420	−226	−809	144	183	−493
60°	404	−247	376	−538	459	−197	116	205	−181	−240	−372	−545
70°	−348	277	34	−446	469	−456	394	−23	−402	448	−712	578
80°	−765	382	144	−676	660	−528	470	−49	−356	608	−1220	730
90°	−156	111	182	−582	821	−815	607	−272	−141	517	−810	693
100°	73	216	134	−406	660	−821	698	−451	224	113	−149	80
110°	−699	456	−146	29	174	−147	−74	−11	42	76	−131	296
120°	−628	661	−494	413	−54	3	643	−359	419	−145	−5	244
130°	−79	458	−532	513	−44	502	59	−259	377	−354	558	−232
140°	−95	378	685	800	−492	342	−105	−342	480	−125	778	−334
150°	604	−18	−380	216	−838	1532	−513	184	259	−493	788	−780

ΔZ

u λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	−749	288	1899	266	−336	−1086	−1761	786	434	−2592	−483	2342
40°	−802	−522	−557	−311	290	300	594	755	155	615	1286	129
50°	456	−388	−827	38	50	−93	−122	−685	−293	76	−890	−574
60°	949	717	500	763	−62	−224	263	−614	−662	425	56	−23
70°	96	−60	−218	−407	146	96	−386	457	799	−243	−227	367
80°	−428	289	246	−420	110	122	−378	251	297	−246	195	46
90°	−509	−46	467	296	−151	−199	111	32	−423	−322	73	−193
100°	−87	−455	−130	342	168	154	449	323	90	53	38	112
110°	664	86	−520	−588	−694	−195	216	−250	−96	364	−26	127
120°	439	236	−29	264	195	−97	−453	−966	−556	254	44	−51
130°	−313	433	102	220	1866	1127	−755	41	634	−264	−274	−297
140°	−752	−315	213	−820	−2189	−1991	170	2015	1153	[illegible]	−137	−85
150°	−384	−105	524	−405	893	2115	−234	−2286	−2282	−1055	1454	1704

ΔZ

u λ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
30°	−605	209	2019	69	−307	−1617	−1345	430	1450	−5637	1196	547
40°	−361	−1210	−161	−974	698	−267	770	795	−346	897	1326	−425
50°	376	−595	131	−1033	177	44	298	−911	−1102	220	−707	−1067
60°	1353	470	876	225	397	−421	379	−409	−843	285	−316	−568
70°	−252	217	−184	−853	615	−360	8	434	397	205	−939	945
80°	−1193	671	390	−1096	770	−406	98	202	−59	362	−1025	776
90°	−665	65	649	−286	670	−1014	718	−240	−564	195	−737	500
100°	−14	−239	4	−64	828	−667	1147	−128	314	165	−111	192
110°	−35	542	−666	−559	−520	−342	142	−261	−54	440	−157	423
120°	−189	897	−523	677	141	−94	190	−1325	−137	109	39	193
130°	−392	891	−430	733	1822	1629	−696	−218	1011	−618	284	−529
140°	−847	63	−472	−20	−2681	−1649	65	1673	1633	−471	641	−419
150°	220	−123	144	−189	55	3647	−747	−2102	−2023	−1546	2242	924

derselben in dem nächstens erscheinenden XVIII. Bande von „Aus dem Archiv der Deutschen Seewarte“ erzeugt und ausserdem durch eine nochmalige Unterstützung die dazu nötige Beschleunigung der noch durchzuführenden Berechnungen ermöglicht hat.

PAULLINIA

SAPINDACEARUM GENUS MONOGRAPHICE DESCRIPTUM.

MONOGRAPHIE

DER

SAPINDACEEN-GATTUNG

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Von

L. Radlkofer.

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